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# ON THE DISTRIBUTION OF INTENSITY IN THE CONTINUOUS SPECTRA OF THE SUN AND THE FIXED STARS, AND ITS RELATION TO SPECTRAL TYPE AND LUMINOSITY

BY

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# Preface.

The present investigation has been performed mainly during the year 1919, the observations and their reduction have been carried out at the Observatory of Upsala.

It is a very pleasant duty to express here my deep sense of obligation to my teacher Professor Östen Bergstrand, who has introduced me to the kind of observations dealt with here as well as to the study of various parts of astronomy, for the great interest he has always paid to my work and for his valuable advice and criticisms. Among the preparatory tasks with regard to the Zeiss-Heyde astrograph which have been useful to me here I wish especially to mention a laborious correction of the position of the lenses performed by Professor Bergstrand and Docent Lundmark.

To Professor Hugo von Zeipel I wish to express here my gratitude for his instruction in practical astronomy and his valuable suggestions concerning questions within the range of the present work. A hypothesis concerning the possibility of deriving absolute magnitudes of stars by means of two color-indices I have for the first time heard clearly expressed by Professor von Zeipel.

To my friend Docent Knut Lundmark I wish to express here my indebtedness for an invaluable, never failing helpfulness in different branches of my work at the Observatory. To other colleagues and friends at the Observatory I am indebted for valuable help during the exposures.

I will also express my thanks here to Mr Henry Alexander, M. A., Lector at the University of Upsala, for a revision of the present paper from the point of view of linguistic correctness.

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*Bertil Lindblad.*

# Introduction.

Already in SECCHI's attempts to classify the stars by means of their spectra attention was paid to the fact that different aspects of the typical spectral lines also defined different states of the spectral energy-curve characterized by the color of the stars. When the laws of STEFAN-BOLTZMANN, WIEN, and PLANCK connecting radiation and temperature became known, interest in the intensity distribution in the continuous spectra was very much increased, the color could now be interpreted to signify different states of temperature of the stellar photospheres, and the series of spectral classes seemed to represent the history of the successive cooling and darkening of the stars. Further investigations on the subject tended to strengthen the correlation between color and spectral type, and it is only recently that we have obtained evidence to show that another very important quality of a star, its absolute magnitude, has an influence upon the color, and thus that color and spectral class are not quite equivalent. In this way it may be of importance to know the exact color even in the case of stars with well known spectral lines. It is especially, however, with regard to the faint stars, whose spectral classes cannot be determined even with the largest instruments, that it becomes a matter of the greatest importance to know more in detail the way in which the two most significant physical qualities of a star, the spectral type and the absolute magnitude, act upon the continuous spectrum, and to what extent those qualities may be determined by examining the spectral energy-curve.

A rigorous determination of the spectral energy-curve has been carried out only in the case of one star, the sun; the most important results concerning the distribution of intensity in the solar spectrum are those of C. G. Abbot[1] and of J. Wilsing[2]. Through these results we know that between wave-lengths 0.47 $\mu$ and 2.0 $\mu$ the solar energy-curve may be well represented by that of a black body of a temperature between 6000° and 7000°. Stefan-Boltzmann's law applied to the value of the solar constant derived by C. G. Abbot, F. E. Fowle, and L. B. Aldrich[3] gives a temperature about 5830°. But when we go to the violet and ultra-violet region, beyond $\lambda$ 0.47 $\mu$, we see that there is a very marked depression of the solar curve in comparison with that of the black body; the change of the curve occurs almost exactly at the limit of the usual photographic spectrum. Thus we cannot immediately apply the laws of black body radiation for the photographic part of the spectrum, which seems to be the most characteristic one concerning the spectral classification of the stars; in all probability the depression of the energy-curve is intimately connected with the very strong accumulation of absorption lines in this region.

For bright stars spectrophotometric investigations have been carried out by several investigators. Determinations of stellar temperatures have been effected in the work of J. Wilsing and J. Scheiner[4], recently continued and revised by J. Wilsing, J. Scheiner, and W. Münch[5], and in the work of H. Rosenberg[6]. The former investigators have used a visual spectrophotometer attached to the 80 cm. refractor at Potsdam. The intensities for several wave-lengths from $\lambda$ 0.448 $\mu$ to $\lambda$ 0.638 $\mu$ are compared with the corresponding intensities of an artificial black body. 199 stars in all have been examined, the temperatures determined according to Planck's law; they range from 2800° to 22500°. The approximation of the energy-curve to that of a black body was very satisfactory for the region examined.

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[1] C. G. Abbot, Astrophys. Journ. 34, p. 197, 1911; Annals of the Astrophys. Obs. of the Smithsonian Inst. III, p. 194, 1913.

[2] J. Wilsing, Publ. des Astrophys. Obs. zu Potsdam, Bd. 22, Nr. 66, 1913; Bd. 23, Nr 72, 1917.

[3] C. G. Abbot, F. E. Fowle, L. B. Aldrich, Annals of the Astrophys. Obs. of the Smiths. Inst. III, pp. 134, 201, 1913.

[4] J. Wilsing und J. Scheiner, Publ. des Astrophys. Obs. zu Potsdam, Bd. 19, Nr. 56, 1909.

[5] J. Wilsing, same publication, Bd. 24, Nr. 74, 1919.

[6] H. Rosenberg, Abh. der Kaiserl. Leop.—Carol. Deutschen Akademie der Naturforscher, Bd. CI, Nr. 2, 1914.

H. ROSENBERG has used a photographic method of research, employing a prism-camera with U.-V.-glass; the diameter of the objective was 11 cm. For the sun and 70 of the most brilliant stars he has measured the intensities relative to certain comparison stars for a great number of points in the spectrum between $\lambda$ 0.345 $\mu$ and $\lambda$ 0.590 $\mu$; for late type stars the measurements do not extend so far in the ultra-violet. The scale of intensity is found by varying the time of exposure. The intensity for a certain wave-length is then expressed by the corresponding mean intensity for stars of MISS MAURY's type II as a unit. If the stars follow the laws of black body radiation, the logarithms of these intensities ought to be a linear function of $\frac{1}{\lambda}$. Now we see from ROSENBERG's figures that this is generally the case, but for the hydrogen stars, MAURY's types IV—X, there is as very marked depression of the curves in the ultra-violet, probably due to the crowding of hydrogen lines beyond $\lambda$ 0.4 $\mu$. A depression of the solar curve in the ultra-violet is also indicated; a corresponding depression is somewhat more conspicuous for $\alpha$ Bootis, somewhat less for $\alpha$ Aurigœ[1]. The temperatures are determined according to PLANCK's law by the difference in magnitudes between the intensities of $\lambda$ 0.400 $\mu$ and $\lambda$ 0.500 $\mu$; they range from $2150^0$ to $\infty^0$. The absolute scale of the temperatures depends on ABBOT's values of the intensities in the solar spectrum for the two wave-lengths in question. Apart from some systematic differences the temperature scales of WILSING and ROSENBERG agree well with each other[2].

The investigations of the solar energy-curve have shown that we cannot strictly speak of a common effective temperature for the whole spectrum of a star (see for instance C. G. ABBOT, The Sun, p. 113). We may, however, take the temperature in any system or, still better, its reciprocal value as a color-equivalent; a comparison between these reciprocal temperatures and the spectral type, which for the bright stars of Wilsing, Scheiner, and Münch and of Rosenberg may generally be taken from the detailed classification of Miss Maury[1], is of great interest, the behavior of this color-equivalent being typical for all color-systems. For the helium and hydrogen stars the color is tolerably constant with an indication of a minimum for class VII, then the color increases, i. e. the temperature falls, with a successively increased gradient through the types with metallic lines; in the following types XVII—XX, characterized by a successively stronger appearance of flutings in the spectrum, the color becomes once more almost constant. For these exceedingly red stars the different systems of temperature fairly well agree in showing temperatures between 2200$^0$ and 3200$^0$.

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[1] This result for $\alpha$ Aurigœ is contrary to what may be expected, the sun being a dwarf and $\alpha$ Aurigœ ia giant of the same spectral class. We must, however, bear in mind that $\alpha$ Aurigœ is a spectroscopic double star with a relatively strong companion of an early type. In the work by WILSING, SCHEINER, and MÜNCH cited the temperature of the star falls out abnormally high for its spectral class. This fact also explains the equality in spectral energy-distribution between $\alpha$ Aurigœ and the dwarf star Groombridge 884 found by W. S. ADAMS (Astrophys. Journ. 39, p. 92).

[2] J. WILSING, loc. cit., p. 3 2; Astr. Nachr. 193, Nr. 4628.

In a special class among the investigations in celestial photometry must be placed the very interesting investigations by W. W. Coblentz[2] concerning the total energy of radiation of the stars. By a very sensitive thermoelectrical method he has measured the radiation for 109 bright stars, using the Crossley reflector of the Lick Observatory. The results have been discussed by K. Burns[3] who has computed the differences between the radiative magnitudes and the visual and photographic magnitudes, thus forming two new color-indices, Rad.-Vis. and Rad.-Phot., which he compares with the spectral type. An application of Planck's law to the values of Rad.-Vis. and Rad.-Phot. leads to a scale of temperatures which may be brought into agreement with that of Wilsing and Scheiner. For the early type stars, however, the inference to be drawn from Rad.-Vis. and Rad.-Phot. is rather vague. For some stars Coblentz has also measured the energy transmitted through a water-cell, for which the coefficient of transmission decreases rapidly towards the infra-red part of the spectrum. When the laws of black body radiation were applied to the results of these measurements, Burns found temperatures in good agreement with those of Wilsing and Scheiner for types A 5—M, but temperatures very much too low for the two stars $\beta$ Orionis and $\alpha$ Lyræ of the types B 8 p and A. It is probable that this discrepancy is caused by the depression of the energy-curves in the ultra-violet for hydrogen stars, this region being of subordinate importance for the visual and photographic magnitudes. Thus it seems to be a too hasty inference to draw the conclusion that the radiation of early type stars does not at all follow PLANCK's law. Another remarkable result of the measurements of COBLENTZ is, according to BURNS, that there seems to be no relation between the absolute total energy of a star and its spectral class. Further it seems as if individual stars may depart rather widely from the mean energy-distribution for a class.

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[1] Harvard Annals 28, part I.

[2] W. W. Coblentz, Lick Observatory Bulletin, 8, p. 104; Bulletin Bur. Stand., 11, p. 613, 1914,

[3] K. Burns, Publ. of the Astron. Soc. of the Pacific. Vol. 27, No. 159, 1915.

A photometric method which may also be used for determinations of color-equivalents is the photo-electric method first applied to stellar photometry by P. GUTHNICK[1]. The method depends on the photo-electric effect of certain metals, that is the emission of electrons by the influence of a beam of light. The magnitudes derived in this way have very great accuracy, further different metals may have widely different maxima of sensibility, and in this way color-indices of stars may be obtained by the use of different cells. The method is of great importance, among other things, for the study of the variability of spectroscopic double stars. Stars of this kind often have a peculiar appearance of the spectral lines; the distribution of energy in the spectrum may also be irregular and variable. Here we have a new quality of a star which may be detected by color-investigations, and to which we must pay attention, if it is not to act as a source of error[2]. According to GUTHNICK[3] abnormal color is perhaps a sure indication of duplicity. Above all he suspects the c stars in MISS MAURY's classification to be spectroscopically double.

Spectrophotometric and radiometric investigations have only been carried out for some of the brighter stars, and even if the instruments are made much more sensitive, in the case of the great majority of stars we shall have to be content with more primitive color-measurements. However, the most primitive and natural of all, the visual color, represented for instance in the excellent catalogues of H. OSTHOFF[4], must also be limited to the relatively bright stars, and is of no great importance for the study of the stellar universe down to faint magnitudes. As a universal color-equivalent we have especially the ordinary color-index, the difference between the photographic and visual or photo-visual magnitude. Among the most important works on this subject are those of K. SCHWARZSCHILD[1], J. PARKHURST[2], and E. S. KING[3]. For the faint stars of stellar clusters color-indices have been determined by H. SHAPLEY[4]. A modification of the process is the method of exposure-ratios invented by F. H. SEARES[5]. Using an orthochromatic plate he determines the times of exposure with and without a yellow screen which give the same impression of a star. The ratio of these times may be used as color-equivalent.

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[1] P. GUTHNICK, Astron. Nachr. 196, Nr. 4701, 1913. P. GUTHNICK und R. PRAGER, Veröff. d. K. Sternwarte zu Berlin—Babelsberg, Bd. 1, Heft 1, 1914.

[2] Cf. the note on $\alpha$ Aurigæ, page 3.

[3] Astron. Nachr. 199, Nr. 4763, 1914.

[4] H. OSTHOFF, Astron. Nachr. Bd. 153, Nr. 3657, 1900; Pubblic. della Specola Vaticana, Serie II, Nr. VIII, Wien 1916.

Another method of obtaining color-equivalents down to very faint magnitudes, on which I have based the empirical results of this paper, is the use of very short grating spectra, the method of determining photographic effective wave-lengths. This method is developed by E. HERTZSPRUNG and Ö. BERGSTRAND[6]; it has been found that the color-equivalent in question is almost analogous to the color-index. Some of the principal properties of the effective wave-lengths are discussed by the writer[7] in a recent paper.

A relation between stellar luminosities and spectral intensity-distribution was at first suspected in connection with the question of the existence of a selective absorption of light in space. A compilation of the most prominent facts indicating the existence of a change of color in some way connected with the distances of the stars has been made by J. C. KAPTEYN[8]. We shall consider here especially some investigations which compare the colors of stars of the same spectral type but with varying distances and absolute magnitudes. For each star we may put an equation

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[1] K. SCHWARZSCHILD, Abh. d. Kgl. Gesellsch. d. Wiss. in Göttingen, Math.-Phys. Klasse, N. F. Bd. VIII, No. 4.

[2] J. PARKHURST, Astrophys. Journ. 36, p. 169, 1912.

[3] E. S. KING, Harvard Annals 59, IV—VI, 1912.

[4] H. SHAPLEY, Studies based on the colors and magnitudes in stellar chesters, Astrophys. Journ., Mt. Wilson Contr.

[5] F. H. SEARES, Proceedings of the National Academy of Sciences of the U. S. of America, 2, p. 521, 1916.

[6] From a methodical point of view the most important works are Ö. BERGSTRAND, Nova Acta R. Soc. Scient. Upsal. Ser. IV, vol. 2, No. 4, 1909; Comptes Rendus 148, 1909; E. HERTZSPRUNG, Publ. des Astrophys. Obs. zu Potsdam, Bd. 22, Nr. 63, 1911.

[7] B. LINDBLAD, Arkiv för Mat., Astr. och Fysik, utgivet av K. Svenska Vetenskapsakademien, Bd. 13, No. 26, 1918.

[8] J. C. KAPTEYN, Astrophys. Journ. 40, p. 187, 1914; Mt. Wilson Contr. No. 83.

$$\text{Observed color-index} = a + b \,.\, m + c \,.\, M + d \,.\, \Delta,$$

where $m$ is the apparent, $M$ the absolute magnitude, the latter being defined as the apparent magnitude at a distance of 10 parsec. $\Delta$ is the distance of the star, $a$ is a constant depending on the spectral type, $b$, $c$, $d$ coefficients to be determined.

J. C. Kapteyn[1] has used bright stars classified by Miss Maury and Miss Cannon, the distances are estimated by means of Kapteyn's empirical formula with arguments apparent magnitude and proper motion. Dropping the term $cM$ he gets $d = +\,0^{\mathrm{m}}.0031 \pm 0.0006$ p. e., the unit of distance being 10 parsec (32.6 light-years). In an analogous investigation P. J. Van Rhijn[2] uses the color-indices of the Yerkes Actinometry. As a mean value for all spectral classes he finds $d = +\,0^{\mathrm{m}}.0015 \pm 0.0003$. This result is ascribed either to an absorption of light in interstellar space or to the influence of the absolute magnitude on the color. There are indications that the phenomenon does not exist for the B and early A stars. Omitting these spectral types Van Rhijn finds for the other $d = +\,0^{\mathrm{m}}.00195 \pm 0.0003$.

The material of the Yerkes Actinometry has also been treated by H. S. Jones[3]; his results, found by combining the spectral classes B8 to K5, are $c = +\,0^{\mathrm{m}}.0336 \pm 0.0053$, and $d = +\,0^{\mathrm{m}}.00473 \pm 0.00035$. Thus the coefficient $d$ is greater than Van Rhijn's value, but is partly compensated by a coefficient $c$ working in the direction to make the more luminous stars bluer than the fainter ones. Different values of the coefficients $c$ and $d$ have been derived by E. S. King[4] from carefully determined color-indices for 28 stars with known parallaxes between the spectral types A5 and K5. His result was $c = -0^{\mathrm{m}}.0254 \pm 0.0030$, $d = +0^{\mathrm{m}}.0189 \pm 0.0065$. The coefficient $d$ is much greater than in the previous authors; $c$ indicates that the more luminous stars are more red than the faint ones. We must conclude that in comparing stars of the same spectral type and of the same apparent magnitude the effect of distance involved in the two coefficients $c$ and $d$ increases considerably from the early A stars to the later types. However, the distribution of the effect between the two causes absolute magnitude and distance must be considered very uncertain on account of the questionable validity of the supposition that the former part of the effect is a linear function of the absolute magnitude, which is the supposition involved in the term $cM$ of the above equation.

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[1] Astrophys. Journ. 30, p. 284. 1909: Mt. Wilson Contr. No. 42.

[2] P. J. Van Rhijn, Inaugural dissertation, Groningen 1915; Summary in Astroph. Journ. 43, p. 36; Mt. Wilson Contr. No. 110.

[3] H. S. Jones, Monthly Notices of the Royal Astron. Soc, Vol. 75, p. 4, 1915.

[4] E. S. King, Harvard Annals 76, N:o. 1, 1916.

A new view of the matter was brought forward in connection with the spectroscopic method of determining stellar parallaxes. W. S. Adams[1] had previously compared pairs of stars of the same spectral types but of widely different proper motions, and had found the more distant stars relatively weaker in the violet; here Adams seems to assume a real effect of distance. In the work of W. S. Adams and A. Kohlschütter[2], however, the effect in question is taken up as a probable criterion of different absolute magnitudes. In this direction the work of Adams is continued by G. S. Monk[3].

Using the parallaxes of W. S. Adams and A. H. Joy[4], and his own method of color-research F. H. Seares[5] has shown that the giants and the dwarfs define different color-series for the late type stars. The same fact was also found by the writer[6], using as color-equivalent the effective wave-length. That the effect was to be found also in the visual estimates of color was discovered by the writer in the same work, combining the colors of Osthoff's first catalogue with the compilation of stellar parallaxes given by O. R. Walkey[7].

From Osthoff's colors I found for the spectral types F5—K0 $c = -0^c.12$, corresponding to $-0^m.053$ in the color-index. From the effective wave-lengths for 26 stars from the catalogue of Adams and Joy I found for the types G 0 to K 5 $c = -0.^{\mu\mu}93$, corresponding to $-0^{m}.052$ in the color-index. At the same time reference was made to E. Hertzsprung's[1] results for the distribution of intensity in the spectrum of the globular cluster M. 3, and to the results of K. Lundmark and B. Lindblad[2] for the effective wave-lengths of some spiral nebulæ and globular clusters. It was found in these works that in the photographic region the distribution of intensity corresponded to the spectral lines, if the relations found for fixed stars were used. The coefficient $d$ thus ought to be vanishingly small owing to the immense probable distances of the objects in question. The investigations of H. Shapley on the color-indices and distances of stars in globular clusters must lead us in the same direction; Shapley has been able to assign a superior limit to the coefficient in question, he finds[3]

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[1] W. S. Adams, Astrophys. Journ. 39, p. 89, 1914; Mt. Wilson Contr. No. 78.

[2] W. S. Adams and A. Kohlschütter, Astrophys. Journ. 40. p. 385, 1914 Mt. Wilson Contr. No. 89.

[3] G. S. Monk, Astrophys. Journ. 44, p. 45, 1916; Mt. Wilson Contr. No. 119.

[4] W. S. Adams and A. H. Joy, Astrophys. Journ. 46, p. 313, 1917; Mt. Wilson Contr. No. 142.

[5] F. H. Seares, Publ. of the Astron. Soc. of the Pacific, 30, p. 122, 1918.

[6] B. Lindblad, loc. cit.

[7] O. R. Walkey, The Journal of the British Astron. Assoc. Appendix to Vol. 27, 1917.

$$d < 0^{m}.00002,$$

a value about a hundredth of that derived by Van Rhijn. A still smaller superior limit of $d$ may be derived from new results with regard to the effective wave-lengths of spiral nebulæ by Lundmark and Lindblad[4]. Using the estimate of the distance of the great nebula in Andromeda given by Lundmark[5]

$$\pi = 0''.0000057$$

and considering the spectral type and effective wave-length of the object, we may put

$$d < 0^{m}.000002.$$

The change of color-index observed in different spectral classes thus being referred to differences in the absolute magnitudes, the question arises as to the cause of this effect. It is probable that the presence of certain elements in the atmosphere of a star, i. e.

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[1] E. Hertzsprung, Astrophys. Journ. 41, p. 10, 1915.

[2] K. Lundmark and B. Lindblad, Astron. Nachr. 205, Nr. 4907; Astrophys. Journ. 46, p. 206, 1917.

[3] H. Shapley, Astrophys. Journ. 49, p. 253: Mt. Wilson Contr. No. 156, p. 5, 1918.

[4] K. Lundmark and B. Lindblad, Astrophys. Journ. 50, p. 376, 1919.

[5] K. Lundmark, Kungl. Svenska Vetenskapsakademiens Handlingar. Bd. 60. No. 8; Preliminary notice in Astron. Nachr. Bd. 209, Nr. 5016, 1919.

the appearance of the corresponding characteristic lines in the spectrum, is intimately connected with the conditions of temperature and pressure in these atmospheres. If we make the hypothesis that each spectral type defines equal temperatures of the stellar photospheres, then the differences in color between giants and dwarfs must be ascribed to the influence of the enveloping cooler layers. It seems probable that at least a part of the effect is caused by a selective scattering or absorption anologous to that producing the depression of the solar energy-curve in the violet region, but strongly increased in the vast atmospheres of the giants. If here the operative cause is a scattering following Lord Rayleigh's law, by which the coefficient of scattering is inversely proportional to the fourth power of the wave-length, the effect is strongly selective and the deformation of the energy-curve different from that caused by a change of temperature. It is probable, however, that the depression in the violet is due to a considerable extent to the crowding of absorption lines in that region. In this case the effect may be still more selective, and the average coefficient of extinction may follow a much higher power of the reciprocal wave-length than the fourth. On the other hand the persistency of the absolute magnitude effect in the visual region of the spectrum may indicate that the effect is a complicated one, and that we have also to deal with a real change of the effective temperature, working through the whole spectrum.

But in this way it may be possible, by examining the energy-curve in different regions, to discriminate between the effects of the two variables, spectral type and luminosity. The color-investigation ought then probably to be extended as far as possible in the ultra-violet region.

This principle I have tried to realize by means of the short grating-spectra by introducing a new color-equivalent, the minimum wave-length $\lambda_{\text{min}}$, which may be easily measured in combination with the previously mentioned effective wave-length $\lambda_c$. This minimum wave-length simply defines the inner limit of the photographic grating-spectrum, a limit which is generally sharp and well defined; it is determined by measuring the minimum distance between the two symmetrical spectra of the first order. The main properties of the new color-equivalent are discussed by the writer in a recent notice[1], and will be further discussed in the latter part of this paper. At least a rough discrimination between giants and dwarfs seems possible by means of $\lambda_c$ and $\lambda_{\min}$ without any reference to the spectral lines.

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[1] B. Lindblad, Astrophys. Journ. 49, p. 289, 1919.

The question of the variation of color with the stellar luminosity has obtained a very valuable contribution in the recent work of F. H. Seares[1]. Using the method of exposure-ratios he has determined the effect for the spectral classes B 5 to M. For the class A 5 and the giants of type F 5 the effect is shown to work in a direction opposite to the usual one, the less luminous stars being found the redder ones. Here we have a cause which may explain the different values of the coefficient $c$ derived by H. S. Jones and E. S. King. For G and K stars the effect seems to increase with the luminosity more rapidly than is indicated by the term $cM$ in Kapteyn's formula, a fact which may explain the large positive value of the coefficient $d$ found by King. For the type M no perceptible difference between giants and dwarfs is found. In the appearance of the grating-spectra, however, there is a very marked difference, which shows that the effective wavelength and the exposure-ratio do not correspond perfectly to each other, the ultra-violet part of the spectrum playing a more important rôle for the former color-equivalent. At last Seares suggests the possibility that by determining two color-indices from different regions we might determine simultaneously spectral type and luminosity.

These are in brief some important facts from which the present investigation has to start. We are to consider from a more theoretical point of view the solar radiation and its energy-curve and try to apply our results to the spectrophotometry of the stars, in order to get a more clear view of the way in which the photospheric temperature and the dimensions of a star are shown in the spectral distribution of energy. The results of this investigation will be compared with new observations, made with partly improved instrumental means, as to the two color-equivalents of the grating-spectra. The results of the first part of the investigation will, of course, also be applicable to another choice of color-equivalents.

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[1] F. H. Seares, Proc. of the National Academy of Sciences of the U. S. of America, Vol. 5, No. 7, 1919.

# I. Solar radiation.

## 1. Principal properties of solar radiation. Theories as to the constitution of the photosphere.

The main features of the solar energy-curve have been mentioned in the introduction; we have now to decide how these characteristic properties are related to other qualities of the solar radiation. The sun is the only star for which we may in a certain sense analyze the radiation by examining different points of the radiating disk. The distribution of the total energy over different zones of the apparent solar disk has been determined by Secchi, Vogel, Langley, Frost; as to the distribution of intensity for different wave-lengths results have been obtained by Vogel, Very, Abbot and Fowle, Schwarzschild and Villiger. The most important results are without doubt those of C. G. Abbot, F. E. Fowle, and L. B. Aldrich[1], they comprise measurements for a great number of wave-lengths between $\lambda\ 0.386\ \mu$ and $\lambda\ 2.097\ \mu$, and are completed by the investigation of K. Schwarzschild and W. Villiger[2] for the wave-length $0.323\ \mu$. For all wave-lengths there is a successive decrease of the intensity, when we go from the center to the limb, the decrease being very marked for the short wave-lengths, and diminishing continually for the longer wave-lengths.

Important conclusions as to conditions of the radiation of the sun may also be drawn from the appearance of the Fraunhofer lines on the different parts of the solar disk. Here the most remarkable fact is the great agreement between the center and the limb, a fact that cannot be understood if we assume the lines to be formed by real absorption in a perfectly dark atmosphere. In such a case the lines ought to be much wider and deeper at the limb. Some differences, however, have been established. G. E. Hale and

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[1] C. G. Abbot, F. E. Fowle, L. B. Aldrich, Annals of the Astrophys. Obs. of the Smithsonan Inst., Vol. III, p. 154, 1913.

[2] K. Schwarzschild and W. Villiger, Astrophys. Journ. 23, p. 284, 1906.

W. S. ADAMS[1] have found three especially characteristic features, which have been summed up by ADAMS[2] in the following way. »First, the great weakening, and in some cases the complete disappearance of the wings shown by many of the lines near the center of the sun. Second a slight widening, which is characteristic of practically all of the lines in the spectrum, accompanied in most cases by a decided modification of their energy-curves. Third, a strengthening and weakening of the lines which in general closely corresponds to that found in sun-spots». There is a decided weakening of the enhanced lines, especially in the violet and ultra-violet region.

At the same time there are also small displacements of the lines when we go from the center to the limb, as well as small changes of the wave-lengths of lines at the solar center relative to terrestrial sources of light, for instance the arc spectrum. According to W. S. ADAMS[3] there is generally a displacement to the red at the limb relative to the center; the displacements at the center also generally take place in this direction. Probably we have here a combination of Doppler effect and pressure-shift. The pressure in the reversing layer has been estimated by JEWELL, MOHLER and HUMPHREYS[4] and by FABRY and BUISSON[5]; they found pressures of about 5 atm. By comparing the solar displacements for lines with great and small pressure-shifts J. EVERSHED and T. ROYDS[6] find pressures of about 3/4 atm. for iron, nickel and titanium. W. H. JULIUS[7] tries to interpret the displacements as being due to a great extent to the anomalous dispersion; he considers the Fraunhofer lines to be formed mainly by anomalous scattering and refraction. One consequence of this ought to be a mutual influence, a slight repulsion, between the components of very narrow pairs of lines. S. ALBRECHT[8] suggests that he has found an effect of this sort; his results, however, are criticized by Ch. E. St. John[1], who states that no perceptible influence of this kind can be detected. At the same time he points out the inconsistency of the previous estimates of pressure in the reversing layer with the actual displacements of various sorts of lines. This shows us the great difficulty in interpreting the matters in question, and it is evident that for the present no certain inference as to the pressure of the reversing layer can be drawn from the observed data.

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[1] G. E. HALE and W. S. ADAMS, Astrophys. Journ. 25, p. 300, 1907; Mt. Wilson Contr. No. 17.

[2] Astrophys. Journ. 31, p. 30, 1910; Mt Wilson Contr. No. 43.

[3] W. S. ADAMS, loc. cit.

[4] L. E. JEWELL, J. F. MOHLER and W. J. HUMPHREYS, Astrophys. Journ. 3, p. 138, 1896.

[5] Ch. FABRY and H. BUISSON, Astrophys. Journ. 31, p. 97, 1910.

[6] J. EVERSHED and T. ROYDS, Kodaikanal Bulletins No. 39, 1914, T. ROYDS, Kodaikanal Bulletins No. 53, 1916.

[7] See for inst. W. H. JULIUS, Astrophys. Journ. 31, p. 419, 1910.

[8] S. ALBRECHT, Astrophys. Journ. 41, p. 333, 1915; 44, p. 1, 1916.

We may try to bring the facts mentioned about the intensity fall along the apparent solar radius into accordance with Young's opinion that the photosphere of the sun is a real surface, caused by a condensation of certain elements into cloud-like formations, enveloped by a cooler, absorbing atmosphere. In this way we may try to determine the coefficient of transmission of the solar atmosphere for different wave-lengths, starting from the values of the distribution of intensity over the solar disk. We find, however, successively increased values of the coefficients when we go from the center towards the edge; as explanations of this fact several causes have been suggested. H. v. Seeliger[2] introduces comparatively strong indices of refraction, increasing for the short wave-lengths, in the atmospheric layers of the sun, F. Very[3] suggests several explanations, among others that the absorbing envelope may have a depth which cannot be considered very small compared with the solar radius. A. Schuster[4], however, shows that the apparent change of the coefficient of transmission may be most simply explained by the assumption that the gaseous envelope has a radiation of its own, which adds to the primary photospheric radiation. In this direction the discussion of the subject must undoubtedly lead us. If there is a real absorption in the atmosphere, the emission must be a radiation of temperature; if we suppose an extinction due to scattering, we must pay attention to the gains of the beam of light by some of the light scattered in all parts of the atmosphere. In the former case the name of photosphere for a certain layer becomes a somewhat fictitious one, and we have left the supposition of the cool atmosphere.

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[1] Ch. E. St. John, Astrophys. Journ. 44, p. 311, 1916.

[2] H. v. Seeliger, Sitzungsber. der K. Bayer. Akademie der Wiss. Math. Phys. Kl., 21, p. 247, 1891.

[3] F. Very, Misc. Scient. Papers of the Alleghany Obs., New Series, No. 9; Astrophys. Journ. 16, p. 73, 1902.

[4] A. Schuster, Astrophys. Journ. 16, p. 320, 1902.

Because of experiences gained at the annular eclipse of April 17, 1912 W. H. Julius[1] now comes to the conclusion that the decrease of brightness from the center towards the limb cannot be ascribed to absorption or scattering in an atmosphere enveloping a body that would otherwise appear uniformly luminous. He finds that the total amount of light radiated and scattered from the entire solar atmosphere is a very small fraction $\left(\text{about } \frac{1}{1000}\right)$ of the solar output. The cause of the intensity fall must thus be sought for in the photosphere itself. In a subsequent paper[2] Julius tries to give an explanation in this direction.

These are some causes which may lead us to favour a view of the solar constitution given by Abbot in his monograph »The Sun», in which he does not accept the condensation theory of the photosphere, but assumes a continually increasing density of purely gaseous matter starting from the thin extreme vapors. We must assume that the distinct boundary shown by the solar limb is due to a rapid increase of density at a certain point of the radius, leading to a very strong increase of the molecular scattering, the absorption, and the radiative power of the vapors. If the density gradient is sufficiently great, for observers on the earth the sun will show a sharp boundary like that of a solid radiating sphere. In this way we may still use the name of photosphere, but as a common name for those layers which contribute most effectively to the radiation. At all events, the transition from photosphere to atmosphere becomes rather vague and hard to define. Schmidt's theory, according to which the sharp solar limb is an optical illusion due to the refraction of the rays in the solar gases, must be rejected as soon as we admit an appreciable general absorption or scattering.

The direct consequence of Abbot's point of view must be that we look into deeper and hotter layers in the direction of the sun's center than near the limb; the decrease of intensity along the apparent radius is thus easily understood. The theory will also have certain consequences for the energy-curve of the sun's center; we have, in fact, a melting together of several curves of different temperatures, corresponding to layers of different depths. If there is, for instance, a strong selective extinction, increasing for the short wave-lengths, the violet light will have a tendency to come from layers higher and cooler than the red light; the consequence must be a distortion of the energy-curve from that of the black body.

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[1] W. H. Julius, Astrophys. Journ. 37, p. 225, 1913.

[2] Astrophys. Journ. 38, p. 129, 1913.

J. WILSING's[1] theory may be considered as a mediative position between those of YOUNG and ABBOT. He retains the hypothesis of the cloudy photosphere, but ascribes no perceptible general absorption or scattering to the overlying atmosphere. The intensity fall along the apparent radius is explained in exactly the same way as in ABBOT's theory; it is, in fact, the explanation suggested by S. ARRHENIUS[2]. The »mean» temperature of the radiation must decrease with increasing distance from the center of the disk, that is with an increasing angle of incidence of the line of sight. At the limb the »mean» temperature must be the same as the true temperature of the photospheric surface.

The most important attempts to give a mathematical treatment of the conditions prevailing in the photospheric and atmospheric layers of the sun have been made by A. SCHUSTER and K. SCHWARZSCHILD. The former[3] puts the problem of determining how the radiation from a plane surface is altered in passing through a »foggy» atmosphere, i. e. an atmosphere which in addition to a real absorption also produces a scattering of the light. SCHUSTER does not consider different angles of inclination of the rays, and he solves the differential equations of the radiation for the two assumptions that the power of emission of the atmosphere is constant through all its height, and that it varies uniformly with the height. In the applications to the sun and the stars SCHUSTER assumes that the continuous spectrum is affected only by scattering, the Fraunhofer lines only by absorption. The appearance of bright and dark lines in the spectrum is related to the ratio of the coefficients of absorption and scattering and to the intensity or the intensity gradient of the atmospheric radiation.

SCHWARZSCHILD[4] has started from somewhat different assumptions in developing the theory of radiative equilibrium. He assumes a continuous increase of density and temperature for the successive layers of the solar envelope; further he assumes in the same way as Schuster that the radiation follows Kirchhoff's law, i. e. that we are concerned with a pure radiation of temperature. The scattering is disregarded, and a common coefficient of absorption for all wavelengths, proportional to the density, is assumed. The layers of equal density are assumed to be parallel, and the gravitation is supposed to be constant through all heights. Schwarzschild now puts the question as to what temperatures are to be ascribed to the different layers, if the immense out-flow of energy passes through the solar atmosphere without changing the conditions of the separate layers, their temperatures and densities. This state is called radiative equilibrium. Using Stefan-Boltzmann's law for the integral radiation, Schwarzschild finds for the temperature $t$ of a layer with pressure $p$,

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[1] J. WILSING, Publ. des Astrophys. Obs. zu Potsdam, Bd. 23, Nr. 72, p. 30.

[2] S. ARRHENIUS, Lehrbuch der kosmischen Physik, I, p. 92, 1903.

[3] A. SCHUSTER, Astrophys. Journ. 21, p. 1, 1905.

[4] K. SCHWARZSCHILD, Nachrichten von der K. Gesellschaft der Wiss. zu Göttingen, Mathem.-phys. Kl., 1906, p. 41.

$$t^4 = \frac{1}{2} T^4 \left(1 + \frac{k}{g} p\right) ,$$

where $T$ is the effective solar temperature, $g$ the acceleration of gravity. The coefficient of absorption is assumed to be $\varrho \,.\, k$, where $\varrho$ is the density.

At the limit of the atmosphere, $p = 0$, we get the temperature

$$\tau = \frac{1}{\sqrt[4]{2}} T .$$

If $T = 6000^0$, $\tau$ comes out as $5050^0$. The extent of the atmosphere is theoretically unlimited. The temperature as a function of the depth of a layer is given by integration of the equation

$$\frac{dp}{p} = \frac{M}{R} \frac{g}{t} dh ,$$

where $M$ is the molecular weight, $R$ the universal gas constant. If the solar atmosphere were made up of ordinary air, an increase of density in the proportion 0.02 to 159.4 would be caused by a difference in depth of 93 km. Because at the distance of the sun 725 km. corresponds to one second of arc, it is evident that the apparent boundary of the sun must be perfectly sharp.

After having proved that the radiative equilibrium is stable for all gases with one, two or three atoms in the molecule, SCHWARZSCHILD applies the theory to the distribution of intensity on the solar disk. If $F(i)$ is the radiation in a direction which makes an angle $i$ with the vertical, SCHWARZSCHILD finds

$$F(i) = \frac{1 + 2 \cos i}{3} .$$

This expression is in much better agreement with the observations than that deduced on the assumption of adiabatic equilibrium.

An investigation which has some features in common with SCHWARZSCHILD's paper referred to above has been carried out by E. ÖPIK[1]. He assumes a linear relation between $t$ and $m$, and applies the principle of equilibrium of radiation to the surface layer of the photosphere. Formulæ are developed giving the intensity distribution along the radius of the solar disk for the integral radiation as well as for the separate wave-lengths.

The theory of radiative equilibrium has recently been applied to the conditions of the interior of a star by A. S. EDDINGTON[2] and J. H. JEANS[3].

In a later paper[4] SCHWARZSCHILD has dealt with the conditions of the solar atmosphere, introducing both absorption and scattering. The formulation of the problem is the same as in SCHUSTER's investigation; the radiation from a black plane surface is assumed to travel through a foggy atmosphere, but SCHWARZSCHILD considers rays of different inclinations. The differential equations of the radiation lead to an integral equation of a rather complicated appearance. For the special case when the coefficient of scattering is zero the solution is given immediately. We assume with SCHWARZSCHILD that the emissive power $E(x)$ of a black body at the same temperature as a layer of depth $x$ below the upper surface of the atmosphere is given by

$$E(x) = a + bx ,$$

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[1] E. ÖPIK, Astron. Nachr. 198, Nr. 4732, 1914.

[2] A. S. EDDINGTON, Monthly Notices 77, p. 16, 1916.

[3] J, H. JEANS, Monthly Notices 78, pp. 28, 36, 1917.

[4] K. SCHWARZSCHILD, Sitzungsberichte der K. Preuss. Akademie der Wiss. 1914, p. 1183.

where $a$ and $b$ are constants. If the height $H$ of the atmosphere and the coefficient of absorption $\varkappa$ are sufficiently great, in order that the quantity $e^{-\varkappa H}$ will be small, we get for the remaining radiation making an angle $i$ with the vertical

$$b(i) = a + \frac{b}{\varkappa} \cos\, i\ .$$

For the other special case when the coefficient of absorption is zero, we get for sufficiently great values of $\sigma H$, $\sigma$ being the coefficient of scattering, approximately

$$b(i) = \frac{0.5 + \cos\, i}{1 + \sigma H}\ .$$

Now SCHWARZSCHILD applies these results to the portion of the spectrum around one of the Fraunhofer lines. In the first case the line must vanish when we approach the solar limb, that is when cos $i$ becomes zero; in the second case both the continuous spectrum and the Fraunhofer line are reduced in intensity in the proportion 1 : 3 at the limb relative to the center.

The persistency of the Fraunhofer lines at the limb, which SCHWARZSCHILD examines by photometric investigation for the lines H and K, seems to favour the latter point of view. The law for the decrease of intensity found here is exactly the same as the function $F(i)$ deduced on the assumption of radiative equilibrium, and fits rather well for the integral radiation. The different behavior of separate wave-lengths, however, is not accounted for by the theory of scattering.

It is evident that the results of SCHUSTER and those of SCHWARZSCHILD's later paper, referred to above, may be applied to the cloudy theory of the solar photosphere, but are, of course, also useful for ABBOT's theory. The agreement with ABBOT's opinions is especially great with regard to the importance of the scattering of light. ABBOT assumes that the radiation coming from and traversing successive gaseous layers is strongly scattered by the vapors according to RAYLEIGH's law, and that it is this scattering that limits our view in the direction of the sun's center and also gives rise to the characteristic distribution of energy over the solar disk.

It seems, however, as if this point of view were not entirely tenable, even if we can explain in some way the different speed of the intensity-fall along the apparent radius for different wave-lengths. It is quite evident that if we assume the solar radiation to be a radiation of temperature, the absorption of the rays in the solar vapors must be of predominant importance. The solar energy-curve may be roughly represented as that of a high-temperated black body, thus according to KIRCHHOFF's law the total power of absorption of those layers from which the radiation that reaches us is emitted approximates to unity. Now there is probably a rather strong scattering in the solar vapors, and consequently at all events those layers can only form a thin shell compared with the whole body of the sun. But then it follows that the coefficient of absorption of the solar gases must also be rather large, at least that it cannot be neglected by the side of the coefficient of scattering in the treatment of the problem. We may therefore conclude that the results in SCHWARZSCHILD's first paper involve a considerable approximation to the truth. An obvious fact to which we may call attention in this connection is that a scattering of light is equivalent to an equilibrium of radiation, not only for the integral radiation as in the case of SCHWARZSCHILD, but for each wave-length separately.

The extinction of light in the terrestrial amosphere is thoroughly discussed by L. V. KING[1]. The coefficient of extinction $C$ ruling the attenuation of the solar radiation he finds to be of the form

$$C = \beta \lambda^{-4} + \gamma ,$$

where the first term is due to scattering. For wave-lengths $\lambda > 0.610\,\mu$ this term is a sum of two, one due to the influence of changes in the density of the air-molecules and following RAYLEIGH's law, the other due to the atmospheric dust. From the run of $C$ with $\lambda^{-4}$ KING concludes that »the ratio of energy scattered to energy absorbed and converted into heat is constant for any wave-length and is independent of the nature of the scattering particle whether »dust» or molecules». For shorter wave-lengths the atmospheric dust will only cause an absorption, the scattering is wholly due to the general radiation of opalescence in gases.

We may imagine the quotient $\frac{\gamma}{\beta}$ to be a function of temperature and pressure; according to the reasoning above it ought to have a considerably greater value for the extremely hot photospheric gases of the sun than for the terrestrial atmosphere. On the basis of WILSING's theory a considerable coefficient of absorption may be ascribed to the small liquid particles forming the clouds of condensation in the photospheric vapors. At all events we assume below that the solar gases have an appreciable coefficient of absorption, and that the radiation follows the laws of KIRCHHOFF and PLANCK. The suitability of this hypothesis is to be shown by its consequences for the observed properties of the radiation.

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[1] L. V. KING, Proceedings of the Royal Society, A, Vol. 88, 1913.

On the other hand we must probably also reckon with a general scattering following RAYLEIGH's law. If the coefficient of scattering is $\sigma$, and the height of the homogenous solar atmosphere is $H$, we have, according to SCHUSTER and SCHWARZSCHILD, for the part of the incident radiation transmitted through the solar atmosphere

$$\frac{1}{1 + \sigma H}$$

For $\sigma H$ we put $m \lambda^{-4}$, where

$$m = \frac{32}{3} \pi^3 \frac{(n - 1)^2}{N} H \, .$$

$n$ is the index of refraction, $N$ the number of molecules pur cubic centimeter. For the terrestrial atmosphere KING found $m = 0.010$ ($\lambda$ in $\mu$).

Now $(n - 1)$, $N$ and $\frac{1}{H}$ are proportional to the density, consequently for the same scattering mass the factor $m$ is independent of the density. If we assume the pressure in the reversing layer to be 1 atm., and imagine the overlying mass compressed to the same density as that of the homogenous atmosphere of the earth, the height $H$ will be $\frac{1}{27.2}$ times the height of the latter. Further, if we assume that the solar atmosphere is built up of dissociated hydrogen, we have $N$ 28.9 times the number of molecules per unit volume air of the same density. For $(n - 1)$ we take the value for undissociated hydrogen; it is nearly 7 times greater than for air of the same density. Under the suppositions made we have therefore for the solar atmosphere,

$$m = 0.00062$$[1]

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[1] This result is at variance with the computation of $m$ made by A. DEFANT, Sitzungsberichte der K. Akademie der Wiss. in Wien, Math.-naturwiss.

and for some wave-lengths

| $\lambda$ | $0.35\,\mu$ | $0.37\,\mu$ | $0.4\,\mu$ | $0.5\,\mu$ | $0.6\,\mu$ |
|---|---|---|---|---|---|
| $\sigma H$ | 0.041 | 0.033 | 0.024 | 0.010 | 0.005 |

Thus the effect of opalescence scattering in those layers where the Fraunhofer lines are formed must probably be vanishingly small, except for the extreme ultra-violet region.

## 2. Radiative equilibrium.

### The distribution of intensity over the solar disk. The constitution of the Fraunhofer lines.

The starting points for the following treatment of the solar radiation are essentially the same as in Schwarzschild's second paper quoted above. The solar atmosphere we consider to be built up of an infinitude of plane layers, which are supposed to have the same temperature and density everywhere. In this way the radiation must be the same at all points of a layer, and we need only consider the radiation as a function of the depth $x$ below a certain surface of reference, dropping the coordinates $y$ and $z$. The $x$-axis is a perpendicular to the layers with the positive direction towards the sun's center. The radiation in unit time of a certain wave-length making an angle $i$ with the $x$-axis, going through an element of surface $ds$ perpendicular to the beam into a solid angle $d\omega$ is

$$a(x,\ i)\,ds\,d\omega \text{ inwards}$$
$$b(x,\ i)\,ds\,d\omega \text{ outwards,}$$

the angle $i$ being reckoned from 0 to $\frac{\pi}{2}$ from the two opposite directions of the normal. On the way $dx$ sec $i$ the radiation $a$ loses

$$\text{by absorption } \varkappa\, a\, dx \sec i$$
$$\text{by scattering } \sigma\, a\, dx \sec i$$

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Kl., Abt. II a, Bd. 125, Heft. 6, 1916, p. 514. Using the figures of Defant from the two columns of this page in the formula on p. 511, I find $m_{th} = 0.0067$ and 0.00056, instead of 0.0406 and 0.0282. The figures in the columns mentioned seem to be correct, and there must have been a miscalculation by Defant in the application to the formula.

$\varkappa$ is the coefficient of absorption, $\sigma$ the coefficient of scattering; they are considered variable from one layer to another. Further we let $E\,ds\,d\omega$ be the radiation in unit time for the wave-length in question at the surface of a black body of the same temperature as the layer of depth $x$.

The amount of energy absorbed in unit time by unit volume of the layer from a radiation with the intensity $E$ at all points would be[1]

$$4\pi\varkappa E$$

But here the energy going into unit volume in unit time would be equal to the power of emission per unit volume of the black body, and therefore, according to KIRCHHOFF's law, $4\pi\varkappa E$ will be the total amount of energy emitted in unit time by unit volume of the gaseous layer.

Now we make the necessary assumption that the cofficient of scattering $\sigma$ works equally in all directions. This is not strictly true from a theoretical point of view, but the assumption is permissible, because at every point of the gaseous mass beams of light will be concentrated which go in directions fairly equally distributed at least over one hemisphere. Then we have for the energy scattered by unit volume

$$\sigma(\int a(x,\ i)d\omega + \int b(x,\ i)d\omega),$$

each integral extended over its respective hemisphere. Because the radiation has symmetry of rotation with regard to the $x$-axis, we may, with SCHWARZSCHILD, put for this expression

$$2\pi\sigma.\ A$$

where

$$A = \int_0^{\frac{\pi}{2}} a(x,\ i)\ \sin\ i\, di + \int_0^{\frac{\pi}{2}} b(x,\ i)\ \sin\ i\, di \quad .\ .\ .\ (1)$$

As we suppose the scattering produced equally in all directions, we shall have for the energy scattered by the element of volume $dv$ into the solid angle $d\omega$

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[1] See M. PLANCK, Vorlesungen über die Theorie du Värmestrahlung; Leipzig, 1919, § 26.

$$2\pi\sigma A \cdot dv \cdot \frac{d\omega}{4\pi} = dv d\omega \frac{\sigma A}{2}$$

The energy emitted in the same solid angle will be

$$dv d\omega \varkappa E .$$

The sum of the energy emitted and scattered from the element of volume $dv$ will thus be

$$dv d\omega \left( \sigma \frac{A}{2} + \varkappa E \right) = dv d\omega J ,$$

and thus $4\pi J$ is the total amount of energy emitted and scattered by a unit volume.

By considering the change of the radiation when we go from the layer $x$ to the layer $x + dx$ we get the differential equations

$$\left\{ \begin{aligned} \cos i \frac{da}{dx} &= -(\varkappa + \sigma) a + J , \\ \cos i \frac{db}{dx} &= \phantom{-}(\varkappa + \sigma) b - J . \end{aligned} \right. \qquad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where

$$J = \frac{\sigma A}{2} + \varkappa E . \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The equations (1) to (3) constitute the equations of the problem. Here $a$ and $b$ are functions of $x$ and $i$, $J$, $\sigma$, and $\varkappa$ are functions of $x$ only.

From Abbot's theory we might perhaps consider $\sigma$ and $\varkappa$ to be simply proportional to the density of the gases; from Wilsing's point of view this is not permissible.

Schwarzschild expresses the function $J$ in an integral equation, which he solves for the special cases $\varkappa = 0$ and $\sigma = 0$. This leads to the results referred to above.

Instead of proceeding this way, we shall try to apply here to the most general case the principle of radiative equilibrium, suggested in Schwarzschild's first paper. We therefore assume that the integral radiation totalled up over all wave-lengths in the spectrum obeys equations of the same form as those deduced above for a small interval of wave-lengths. For the integral radiation we use the symbols $\bar{a}(x, i)$, $\bar{b}(x, i)$, $\bar{\varkappa}$, $\bar{\sigma}$ etc. Because $\varkappa$ and $\sigma$ are functions of $\lambda$, we must suppose that the composition of the radiation does not alter very much with $i$. As mentioned before, we mean by radiative equilibrium that the extreme solar layers have such temperatures that the continuous outflow of energy does not alter the temperatures and densities of the separate layers. Here we exclude the possibility of a convection of heat and assume the energy to be only propagated by radiation. If the extreme layers of the sun do not share the vertical streamings on which some solar theories insist, it seems very probable that an equilibrium in accordance with this principle is brought about for the photospheric and atmospheric layers.

The radiative equilibrium now demands that for a given arbitrary volume the absorbed energy from the radiations $\bar{a}$ and $\bar{b}$ will be equal to the emitted temperature radiation. Thus, if we consider the absorption and emission per unit volume, we shall have.

$$2\pi\bar{\varkappa}\left(\int_0^{\frac{\pi}{2}}\bar{a}(x, i)\,\sin\, i\, di + \int_0^{\frac{\pi}{2}}\bar{b}(x, i)\,\sin\, i\, di\right) = 4\pi\bar{\varkappa}\bar{E}$$

Thus we get as a condition for the radiative equilibrium

$$\bar{A} = 2\bar{E},$$

and then

$$\bar{J} = (\bar{\varkappa} + \bar{\sigma})\frac{\bar{A}}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Before we apply this relation we make the simplifying assumption that $\bar{a}(x, i)$ and $\bar{b}(x, i)$ are independent of $i$, so that we may drop $i$ in the symbols. Thus we have

$$A = \bar{a}(x) + \bar{b}(x), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1^*)$$

and by multiplying the original differential equations for the integral radiation by sin $i$ and integrating from 0 to $\frac{\pi}{2}$, we get

$$\left\{\begin{aligned} \frac{1}{2}\frac{d\overline{a}}{dx} &= -(\overline{\varkappa} + \overline{\sigma})\overline{a} + \overline{J} \\ \frac{1}{2}\frac{d\overline{b}}{dx} &= \phantom{-}(\overline{\varkappa} + \overline{\sigma})\overline{b} - \overline{J} \end{aligned}\right. \qquad \ldots\ldots \quad (2^*)$$

By adding the relation

$$\overline{J} = \frac{\overline{\sigma}}{2}\overline{A} + \overline{\varkappa}\overline{E} \qquad \ldots\ldots \quad (3^*)$$

we have the new set of equations complete.

If now we make use of the expression (4) for $\overline{J}$ in equations (2*), we get by adding and subtracting,

$$\left\{\begin{aligned} \frac{d}{dx}(\overline{a} + \overline{b}) &= 2(\overline{\varkappa} + \overline{\sigma})(\overline{b} - \overline{a}) \\ \frac{d}{dx}(\overline{a} - \overline{b}) &= 0 \end{aligned}\right.$$

If we put

$$dm = (\overline{\varkappa} + \overline{\sigma})dx\,, \qquad \ldots\ldots \quad (5)$$

we get

$$\left\{\begin{aligned} \frac{d}{dm}(\overline{a} + \overline{b}) &= 2(\overline{b} - \overline{a}) \\ \frac{d}{dm}(\overline{a} - \overline{b}) &= 0 \end{aligned}\right.$$

Thus we have

$$\left\{\begin{aligned} \overline{b} - \overline{a} &= c_1 \\ \overline{a} + \overline{b} &= 2c_1\, m + c_2\,, \end{aligned}\right.$$

where $c_1$ and $c_2$ are arbitrary constants. Now for $m = 0$ we have $\overline{a} = 0$, $\overline{b} = \overline{b}(0)$, and we get

$$c_1 = c_2 = \overline{b}(0)$$

For the radiative equilibrium we have, according to the foregoing,

$$\overline{a} + \overline{b} = 2\overline{E}\,.$$

Consequently we get

$$\left\{\begin{aligned} \bar{a} &= \bar{b}(0)m, \quad . \quad . \\ \bar{b} &= \bar{b}(0)(1+m), \\ \bar{E} &= \frac{\bar{b}(0)}{2}(1+2m), \end{aligned}\right. \qquad . \; . \; . \; . \; . \; . \; . \quad (6)$$

For $\bar{E}$ we apply Stefan's law $\bar{E} = c\,t^4$, $t$ being the absolute temperature of the layer. $\pi\bar{b}(0)$ is a constant, measuring the total amount of energy radiated into space. In Schwarzshilds first paper it is the quantity designed by $A_0$. If we introduce the temperature $\tau$ of the extreme atmospheric vapors, and put $A_0 = cT^4$, where $T$ is the effective temperature, we have

$$\tau = \frac{T}{\sqrt[4]{2}} \,.$$

Further we get for the temperature of a layer

$$t = \tau\sqrt[4]{1+2m} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

where according to (5)

$$m = \int_{-\infty}^{x} (\bar{\varkappa} + \bar{\sigma})dx$$

Equation (7) gives the temperature of a layer in terms of the optical mass $m$ overlying it, and this relation may be used in the original equations in order to get approximately the amount of radiation for different angles of inclination. It is at once evident that our equations for the integral radiation are of the same form and that our manner of proceeding is here the same as in Schwarzschild's treatment of »Schuster's approximation» in the case of a pure scattering.

We shall now first derive the expression for the part of the integral radiation that forms an angle $i$ with the $x$-axis. For this purpose we use the original equations (2), but applied to the integral radiation, combined with the relation (4), according to which we have

$$\bar{J} = (\bar{\varkappa} + \bar{\sigma})\bar{E}, \quad . \; . \; . \; . \; . \; . \; . \; . \quad (4a)$$

and thus

$$\begin{cases} \cos i \dfrac{d\bar{a}}{dm} = -\bar{a} + \bar{E} \\ \cos i \dfrac{d\bar{b}}{dm} = \quad \bar{b} - \bar{E} \end{cases}$$

If the radiation $\bar{b}(m, i)$ has the value $B$ for $m = M$, we have, by solving the second equation,

$$\bar{b}(m, i) = Be^{(m-M)\sec i} + e^{m \sec i} \int_m^M \bar{E} e^{-m \sec i} \, dm \sec i .$$

Now we put $M = \infty$, whereby the first term vanishes, and apply this relation for $m = 0$. Then we have

$$\bar{b}(0, i) = \int_0^\infty \bar{E} e^{-m \sec i} \, dm \sec i ,$$

where we are to introduce the value of $\bar{E}$ from equation (6). Consequently

$$\bar{b}(0, i) = \frac{\bar{b}(0)}{2} \int_0^\infty (1 + 2m) \, e^{-m \sec i} \, dm \sec i ,$$

$$\bar{b}(0, i) = \frac{\bar{b}(0)}{2} (1 + 2 \cos i)$$

Thus we have found the same expression for the distribution of the integral spectral energy as was given in Schwarzschild's first paper; if we put the energy at the center equal to 1, we have the Intensity for the angle $i$

$$F(i) = \frac{1 + 2 \cos i}{3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Further because the results obtained above are perfectly analogous to those derived in »Schuster's approximation» for a pure scattering, we may apply here to our equations (6) and (8) the corrections to »Schuster's approximation» computed by Schwarzschild. We thus correct our theoretical values of $F(i)$ by means of SCHWARZSCHILD's table (second paper, p. 1197), where we use the values for $H = 8$.

The correspondence between theory and observation is shown in table 1. $\varrho$ is the distance of a zonical element from the center of the sun, expressed as a fraction of the radius. Evidently we have $\varrho = \sin i$. The observed quantities are derived in the following way. ABBOT, FOWLE and ALDRICH[1] have constructed a table of the decrease of intensity for different wave-lengths along the apparent solar radius, where the values for $\varrho = 0$ are adjusted to correspond to the normal energy-curve of the radiation. By adding the values for equidistant wave-lengths belonging to the same zone, — for the infra-red wave-lengths the intensities must be multiplied by certain factors in order to account for the greater size of the intervals —, we get numbers characterizing the integral radiation. The values derived agree well with those of MÜLLER[2].

Table 1.

| $\varrho$ | 0.00 | 0.40 | 0.55 | 0.65 | 0.75 | 0.825 | 0.875 | 0.92 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|
| Observed | 1.00 | 0.95 | 0.91 | 0.87 | 0.82 | 0.78 | 0.71 | 0.66 | 0.60 |
| Computed | 1.00 | 0.94 | 0.90 | 0.86 | 0.79 | 0.73 | 0.69 | 0.62 | 0.57 |
| Difference | 0 | + 1 | + 1 | + 1 | + 3 | + 5 | + 2 | + 4 | + 3 |

The computed values of $F(i)$ are systematically somewhat smaller than those observed; we shall return later to an explanation of this discrepancy. An adiabatic equilibrium would lead to a much more rapid decrease[3].

Our main purpose is, however, to get the law of distribution over the solar disk for each wave-length separately. For this purpose we must examine how far a formula analogous to the relation (4*a*) holds true for the separate wave-lengths. The gaseous or cloudy layers are arranged according to the demands of the integral radiation; because the relation between emissive power and optical mass in (6) is not valid for all separate wave-lengths, the radiative equilibrium cannot be fulfilled for them all. But because the temperature only varies slightly with the optical mass, there will be a certain medium wave-length, not far from the wave-length of maximum energy, for which we may assume the black body emission $E$ to be nearly proportional to the fourth power of the temperature, and for which consequently a relation $E = c(1 + 2m)$ holds true. If the wave-length of maximum radiation lies at $\lambda$ 0.4 $\mu$, corresponding to a temperature of about 7000°, according to PLANCK's law the wave-length in question will lie about 0.5 $\mu$. We have derived above $\bar{E}(0) = \frac{\bar{b}(0)}{2}$ for the integral radiation. Now $\bar{E}$ is according to the theory a black body radiation; from observations we know that $\frac{\bar{b}(0)}{2}$ may be roughly represented by a black body energy-curve, and consequently we may put $E(0) = \frac{b(0)}{2}$ for the medium wave-length. Further we have approximately for this wave-length $\varkappa + \sigma = \bar{\varkappa} + \bar{\sigma}$, provided that $\varkappa + \sigma$ does not vary too fast with the wave-length. If we define

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[1] loc. cit.

[2] G. MÜLLER, Photometrie der Gestirne, p. 323, 1897.

[3] See SCHWARZSCHILD's first paper cited above.

$$\mu = \int_{-\infty}^{x} (\varkappa + \sigma)\,dx,$$

we have in this case almost $\mu = m$. Thus we may put here $E = \frac{b(0)}{2}(1 + 2\mu)$. Then the equilibrium of radiation may be considered as being produced for a part of the spectrum enclosing the wave-length in question in such a way that we may put, with some approximation to the truth, for each separate wave-length of the interval

$$A = 2E,$$

avd thus, if $\sigma$ is not great compared with $\varkappa$

$$J = (\varkappa + \sigma)E \quad . \; . \; . \; . \; . \; . \; . \; . \quad (4b)$$

Moreover, this relation will be very nearly true for all wave-lengths where $\sigma$ is very small compared with $\varkappa$. According to the previous discussion we may therefore expect a fair approximation to the real facts by assuming the relation $(4b)$ to be generally true.

We then have in the same way as for the integral radiation

$$\begin{cases} \cos i \dfrac{da}{d\mu} = -a + E \\ \cos i \dfrac{da}{d\mu} = \phantom{-a+{}} b - E; \end{cases}$$

and thus

$$b(0,\ i) = \int_0^\infty E e^{-\mu \sec i}\, d\mu \sec i.$$

For $E$ we introduce the law of Planck,

$$E = C\lambda^{-5}(e^{\frac{c}{\lambda t}} - 1)^{-1},$$

for $t$ we use equation (7)

$$t = \tau\sqrt[4]{1 + 2m}\,.$$

Nov we put the quotient

$$\frac{dm}{d\mu} = \frac{\overline{\varkappa} + \overline{\sigma}}{\varkappa + \sigma} = k$$

We assume that $\varkappa + \sigma$ does not vary very fast with the wavelength. Because the temperature increases slowly with $m$, the composition of the integral radiation does not change rapidly, and we may suppose $k$ to be constant for the successive layers, and thus have

$$m = k\mu\,.$$

We put

$$\mu \sec i = \xi\,,$$

and thus have

$$b(0, i) = C\lambda^{-5}\int_0^\infty \frac{e^{-\xi}}{e^{\frac{c}{\lambda\tau}(1+2k\xi\cos i)^{-\frac{1}{4}}} - 1}\, d\xi \quad . \quad . \quad . \quad . \quad (9)$$

If we make an assumption about $k$, this relation only contains one unknown parameter $\tau$, which gives immediately, according to (7), the true temperature $t$ of a layer in terms of the optical mass $m$ over it. By studying the value of $b(0, i)$ for different angles $i$, i. e. for different points of the apparent solar radius, we may determine the limiting teperature $\tau$ for the sun,

*In this way the temperature of the successive radiating layers as a function of the overlying optical mass, as well as the effective temperature of the total energy radiated into space and the spectral energy distribution of this radiation may be determined by the decrease of radiation for an arbitrary wave-length along the radius of the solar disk.*

Below we shall apply this principle to observational data of different kinds. The dependence of $\tau$ on the values of $t$ and $m$ for a certain layer will be of fundamental use to us, when we pass to the comparison of the radiations of stars of the same spectral type but of different luminosities.

There is an important remark to be made here as to the influence of the correction to »SCHUSTER's approximation». We see from SCHWARZSHILD's table (p. 1196) that for a large total depth of the scattering layers (in our case the layers in radiative equilibrium) the correction to be applied to $(H + 1) \cdot J(x)$, which quantity corresponds to $\frac{\bar{E}}{\bar{b}(0)}$ in our equations, is a small positive quantity, decreasing with increasing values of $x$. It is therefore evident that in our case the correction in question will probably mainly concern the radiation of the long wave-lengths, because their most effective layers are placed at much higher levels than those of the short wave-lengths, if the coefficient of extinction is the same. As far as this condition is nearly fulfilled we may therefore expect an especially close correspondence to our theory for the short wave-lengths.

The integral in equation (9) may be developed in a series of integrals, which are special cases of the general integral

$$\int_{y}^{\infty} e^{-x\left(t \pm t^{-\mu}\right)} t^{\nu} dt\,.$$

This integral has been developed by H. FAXÉN[1]. Except the functions log $x$ and log $y$ the development contains series of powers in $x$ and $y$, converging for $\mu > -1$.

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[1] Mr. FAXÉN has kindly informed me about his results, wich are still unpublished.

These series, however, are not convenient for a numerical computation in this special case, and therefore I have used here a numerical method of integration. We write (9)

$$b(0,\ i) = K_{\vartheta,\,\varepsilon} \cdot C\lambda^{-5}\left(e^{\frac{c}{\lambda\tau}(1+2k\cos i)^{-\frac{1}{4}}}-1\right)^{-1}, \quad . \quad . \quad (10)$$

where

$$K_{\vartheta,\,\varepsilon} = \int\limits_{0}^{\infty}\frac{e^{\vartheta}-1}{e^{\vartheta\sigma}-1}\,e^{-\xi}d\xi\,,$$

$$\theta = \frac{c}{\lambda\tau}(1+\varepsilon)^{-\frac{1}{4}}\,,\quad \sigma = \left(\frac{1+\varepsilon}{1+\varepsilon\xi}\right)^{\frac{1}{4}},\quad \varepsilon = 2k\ \cos\ i$$

By putting $e^{-\xi} = \eta$ we get

$$K_{\vartheta,\,\varepsilon} = \int\limits_{0}^{1}\frac{e^{\vartheta}-1}{e^{\vartheta\sigma}-1}\,d\eta\,,\quad \sigma = \left(\frac{1+\varepsilon}{1+\varepsilon\,|\,lg\ \eta\,|}\right)^{\frac{1}{4}}.$$

Using Simpson's rule for the part of the integral between 0.1 and 0.2, and between 0.2 and 1, with interwals equal to 0.05 and 0.1 respectively, and a graphical method for the remaining part, I have calculated $K_{\vartheta,\,\varepsilon}$ for some values of $\theta$ and $\varepsilon$ of the order of magnitude which may occur in the following. The results are shown in table 2. The last decimal place is uncertain. By means of the numbers in the table the curves in fig. 1 were constructed, giving $K$ as a function of $\theta$ for different values of $\varepsilon$. The values in italics are read from these curves. We see that the integral is nearly equal to unity for not too large values of $\theta$ and $\varepsilon$ and has a minimum for each curve between $\theta = 0$ and $\theta = 5$. For $\theta = 5.05$ the integral is very nearly equal to unity for all values of $\varepsilon$ in the table. For $\theta = 0$ we get the integral after developing in the first expression on this page.

Table 2.

| $\varepsilon$ \ $\theta$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 8 |
|---|---|---|---|---|---|---|---|---|
| 0.5 | 0.9919 | *0.9876* | 0.9842 | *0.9841* | *0.9884* | *0.9993* | 1 0157 | — |
| 1.0 | 0.9817 | 0.9724 | 0.9661 | 0.9663 | 0.9763 | 0.9986 | 1.0333 | 1.153 |
| 1.5 | 0.9736 | *0.9601* | 0.9510 | *0.9520* | *0.9677* | *0.9985* | 1.0480 | — |
| 2.0 | 0.9672 | 0.9508 | 0.9400 | 0.9414 | 0.9612 | 0.9984 | 1.0602 | 1.260 |
| 2.5 | 0·9619 | — | — | — | — | — | — | — |

We shall now compare our theoretical formula (10) with the observed energy-distribution along the apparent solar radius. In the preceding discussion we have made the supposition that $\varkappa + \sigma$ generally does not vary very fast with the wave-length, further, the relation (4 b) is more reliable the smaler $\sigma$ is compared with $\varkappa$. If the coefficient of scattering $\sigma$ obeys Rayleigh's law, and $\varkappa$ is assumed to be nearly constant for all wave-lengths, the two suppositions are obviously equivalent; they may be simply expressed by the condition that $k$ shall approximate to unity. Thus to begin with, we put $k = 1$, and have

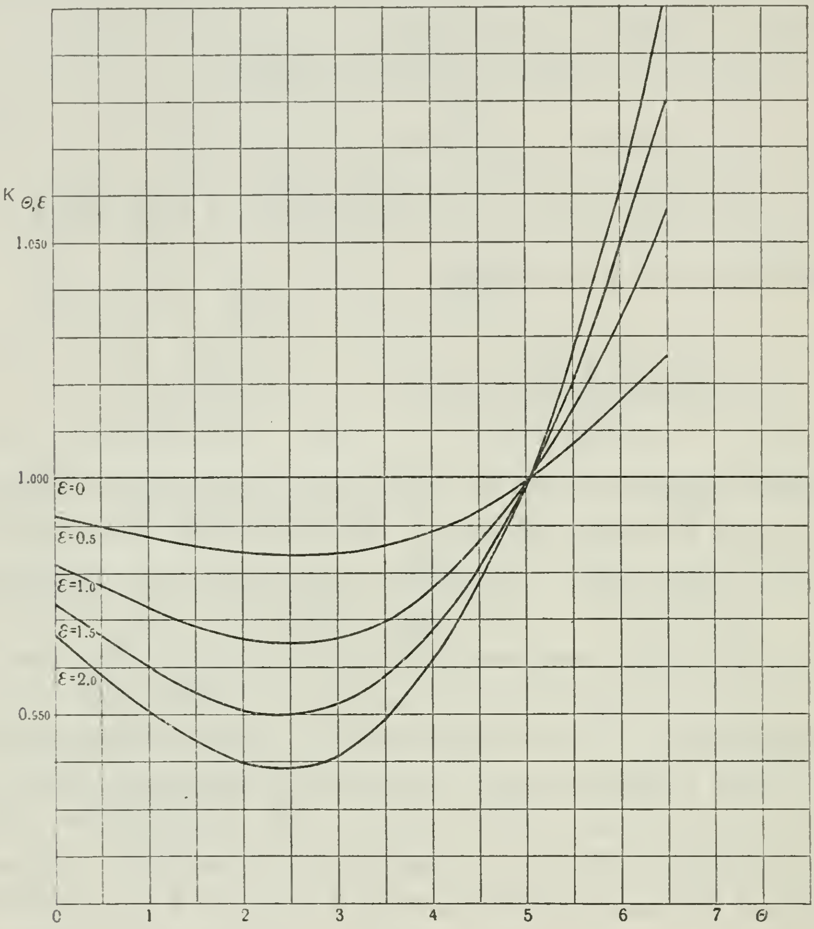

Fig. 1.

$$b(0,\ i) = KC\lambda^{-5}\left(e^{\frac{c}{\lambda\tau}(1+2\cos i)^{-\frac{1}{4}}} - 1\right)^{-1} \quad . \quad . \quad . \quad . \quad (10^*)$$

For the center of the sun we have

$$b(0,0) = K_0 C\lambda^{-5}\left(e^{\frac{c}{\lambda\tau\sqrt[4]{3}}} - 1\right)^{-1}$$

We put

$$\frac{c}{\lambda\tau\sqrt[4]{3}} = \alpha \,, \quad \left(\frac{1+2\cos i}{3}\right)^{-\frac{1}{4}} = M \,.$$

Thus we have, if we define $f(i) = \frac{b(0,\, i)}{b(0,\, 0)}$.

$$f(i) = \frac{K}{K_0}\,\frac{e^{\alpha}-1}{e^{\alpha M}-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The observed values of $f(i)$ are taken from the results of the bolometric work of ABBOT, FOWLE and ALDRICH, completed for wave-length 0.323 $\mu$ by the results of SCHWARZSCHILD and VILLIGER, obtained by photographing the sun by means of a silvered lens. The intensities are tabulated by ABBOT, FOWLE and ALDRICH for nine points of the radius, the intensity at the center being taken as the unit. If the solar atmosphere is supposed to be very thin, the distance $\varrho$ of a zone from the center immediately gives $\varrho = \sin i$; $M$ may thus be calculated. Further we have for the items $\theta$ and $\varepsilon$ of $K$ and $K_0$,

for the center, $\theta_0 = \alpha$, $\varepsilon_o = 2$,
for the distance $\varrho$, $\theta = \alpha M$, $\varepsilon = 2 \cos i$.

Thus if we put for $f(i)$ one of the values observed, we get an equation which only contains the one unknown quantity $\alpha$. For determining $\alpha$ I have chosen $f(i)_{obs}$ for $\varrho = 0.825$, except for $\lambda$ 0.386 $\mu\mu$, where I have used the value for $\varrho = 0.92$, because the value of $\alpha$ thus derived gives considerably less systematic residuals in table 4. We may write equation (11)

$$e^{\alpha(1-M)} = f(i)\,\frac{K_0}{K}\,\frac{1-e^{-\alpha M}}{1-e^{-\alpha}} = f(i)\cdot\Phi \,.$$

When $\alpha$ is great, which is the case for small wave-lengths, we may put $\Phi = 1$ in the first approximation. When a preliminary value of $\alpha$ has thus been derived, a more exact value of $\Phi$ may be calculated, and so on. When $\alpha$ is small, this arrangement will be less expeditious, but if we have got a good approximate value of $\alpha$, we may determine the correction by differentiating equation (11). In this way I have determined $\alpha$ for the sequence of wave-lengths in Abbot and Fowle's table. From the expression for $b(0, 0)$ it is seen that $T_0 = \tau \sqrt[4]{3}$ may be called the apparent effective temperature for the sun's center, in what concerns the relative distribution of energy for a sequence of wave-lengths where $K$ may be regarded as constant. Because

$$\tau \sqrt[4]{3} = \frac{c}{a\lambda},$$

we see that if our assumptions are true, $a\lambda$ ought to be constant; if $c = 14300$, $T_0$ about $7000^0$, we get $a\lambda = 2.0$.

In table 3 the values of $a$ and $a\lambda$ are tabulated. Table 4 shows the observed and computed values of $f(i)$ for the different points of the radius. The computed values are calculated according to equation (11) with the respective values of $a$ in table 3. The differences $\Delta$ are expressed in units of the fourth decimal. Fig. 2 shows $a\lambda$ as a function of $\lambda$ between $0.323\,\mu$ and $1.225\,\mu$.

Table 3.

| $\lambda$ | $0.323\,\mu$ | $0.386\,\mu$ | $0.433\,\mu$ | $0.456\,\mu$ | $0.481\,\mu$ | $0.501\,\mu$ | $0.534\,\mu$ | $0.604\,\mu$ | $0.670\,\mu$ | $0.699\,\mu$ | $0.866\,\mu$ | $1.031\,\mu$ | $1.225\,\mu$ | $1.655\,\mu$ | $2.097\,\mu$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $a$ | 5.800 | 5.325 | 5.024 | 4.460 | 4.139 | 3.990 | 3.709 | 3.206 | 2.805 | 2.700 | 2.065 | 1.700 | 1.428 | 0.699 | 0.378 |
| $a\lambda$ | 1.87 | 2.06 | 2.18 | 2.03 | 1.99 | 2.00 | 1.98 | 1.94 | 1.88 | 1.89 | 1.79 | 1.75 | 1.75 | 1.16 | 0.79 |

Table 4.

| $\varrho$ | 0.00 | 0.20 | 0.40 | 0.55 | 0.65 | 0.75 | 0.825 | 0.875 | 0.92 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\lambda$ 0.323 Obs. . | 1.000 | 0.960 | 0.897 | 0.835 | 0.775 | 0.690 | 0.600 | 0.530 | 0.452 | 0.382 |
| Comp. | 1.000 | 0.981 | 0.923 | 0.847 | 0.776 | 0.687 | 0.600 | 0.528 | 0.448 | 0.380 |
| $\Delta$ | 0 | − 210 | − 260 | − 120 | − 10 | + 30 | 0 | + 20 | + 40 | + 20 |
| $\lambda$ 0.386 Obs. . | 1.0000 | 0.9801 | 0.9258 | 0.8556 | 0.7920 | 0.7097 | 0.6326 | 0.5543 | 0.4826 | 0.4177 |
| Comp. | 1.0000 | 0.9830 | 0.9284 | 0.8594 | 0.7950 | 0.7110 | 0.6286 | 0.5599 | 0.4828 | 0.4168 |
| $\Delta$ | 0 | − 29 | − 26 | − 38 | − 30 | − 13 | + 40 | − 56 | − 2 | + 9 |

| $\varrho$ | 0.00 | 0.02 | 0.40 | 0.55 | 0.65 | 0.75 | 0.825 | 0.875 | 0.92 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|
| λ 0.433 Obs. . | 1.0000 | 0.9780 | 0.9271 | 0.8663 | 0.8060 | 0.7290 | 0.6471 | 0.5827 | 0.5098 | 0.4499 |
| Comp. | 1.0000 | 0.9841 | 0.9333 | 0 8674 | 0.8064 | 0.7261 | 0.6472 | 0.5810 | 0 5056 | 0.4410 |
| △ | 0 | − 61 | − 52 | − 11 | − 4 | + 29 | − 1 | + 17 | + 42 | + 89 |
| λ 0.456 Obs. . | 1.0000 | 0.9857 | 0.9416 | 0.8848 | 0.8314 | 0,7564 | 0.6810 | 0.6160 | 0.5377 | 0.4706 |
| Comp. | 1.0000 | 0.9859 | 0.9405 | 0.8813 | 0.8262 | 0.7532 | 0.6812 | 0.6197 | 0.5491 | 0.4876 |
| △ | 0 | − 2 | + 11 | + 35 | + 52 | + 32 | − 2 | − 37 | − 114 | − 170 |
| λ 0.481 Obs. . | 1.0000 | 0.9871 | 0.9438 | 0.8914 | 0.8401 | 0.7706 | 0.7007 | 0.6378 | 0.5658 | 0.4988 |
| Comp. | 1.0000 | 0.9866 | 0.9445 | 0.8892 | 0.8379 | 0.7694 | 0.7008 | 0.6423 | 0.5746 | 0.5153 |
| △ | 0 | + 5 | − 7 | + 22 | + 22 | + 12 | − 1 | − 45 | − 88 | − 165 |
| λ 0.501 Obs. . | 1.0000 | 0.9850 | 0.9454 | 0.8945 | 0.8449 | 0.7773 | 0.7106 | 0.6501 | 0 5826 | 0 5171 |
| Comp. | 1.0000 | 0.9871 | 0.9467 | 0.8933 | 0.8442 | 0.7770 | 0.7105 | 0.6532 | 0.5868 | 0.5287 |
| △ | 0 | − 21 | − 13 | + 12 | + 7 | + 3 | + 1 | − 31 | − 42 | − 116 |
| λ 0.534 Obs. . | 1.0000 | 0.9870 | 0.9499 | 0.9018 | 0.8561 | 0.7919 | 0.7282 | 0.6720 | 0.6053 | 0.5478 |
| Comp. | 1.0000 | 0.9882 | 0.9503 | 0.9005 | 0.8537 | 0.7911 | 0.7280 | 0.6738 | 0.6102 | 0.5541 |
| △ | 0 | − 12 | − 4 | + 13 | + 24 | + 8 | + 2 | − 18 | − 49 | − 63 |
| λ 0.604 Obs. . | 1.0000 | 0.9894 | 0.9568 | 0.9129 | 0.8722 | 0.8160 | 0.7606 | 0.7102 | 0 6485 | 0.5935 |
| Comp. | 1.0000 | 0.9899 | 0.9572 | 0.9132 | 0.8724 | 0.8171 | 0.7606 | 0.7104 | 0.6534 | 0.6001 |
| △ | 0 | − 5 | − 4 | − 3 | − 2 | − 11 | 0 | − 2 | − 49 | − 66 |
| λ 0.670 Obs. . | 1.0000 | 0.9906 | 0.9612 | 0.9241 | 0.8870 | 0.8376 | 0.7860 | 0.7404 | 0.6803 | 0.6292 |
| Comp. | 1.0000 | 0.9910 | 0.9620 | 0.9226 | 0.8863 | 0.8366 | 0.7859 | 0.7407 | 0.6882 | 0.6404 |
| △ | 0 | − 4 | − 8 | + 15 | + 7 | + 10 | + 1 | − 3 | − 79 | − 112 |
| λ 0.699 Obs. . | 1.0000 | 0.9902 | 0.9629 | 0.9261 | 0.8903 | 0.8412 | 0.7925 | 0.7476 | 0.6909 | 0.6371 |
| Comp. | 1.0000 | 0.9913 | 0.9636 | 0.9253 | 0.8902 | 0.8419 | 0.7924 | 0.7488 | 0.6973 | 0.6505 |
| △ | 0 | − 11 | − 7 | + 8 | + 1 | − 7 | + 1 | − 12 | − 64 | − 134 |
| λ 0.866 Obs. . | 1.0000 | 0.9922 | 0.9690 | 0.9388 | 0.9108 | 0.8711 | 0.8302 | 0.7918 | 0.7440 | 0.6987 |
| Comp. | 1.0000 | 0.9930 | 0.9701 | 0.9391 | 0.9103 | 0.8706 | 0.8302 | 0.7939 | 0.7501 | 0.7103 |
| △ | 0 | − 8 | − 11 | − 3 | + 5 | + 5 | 0 | − 21 | − 61 | − 116 |
| λ 1.031 Obs. . | 1.0000 | 0.9978 | 0.9774 | 0.9508 | 0.9251 | 0.8886 | 0.8507 | 0.8159 | 0,7724 | 0.7298 |
| Comp. | 1.0000 | 0.9937 | 0.9740 | 0.9469 | 0.9214 | 0.8864 | 0.8507 | 0.8187 | 0.7795 | 0.7438 |
| △ | 0 | + 41 | + 34 | + 39 | + 37 | + 22 | 0 | − 28 | − 71 | − 140 |
| λ 1.225 Obs. . | 1.0000 | 0.9948 | 0.9756 | 0.9530 | 0.9316 | 0.9007 | 0.8654 | 0.8336 | 0.7944 | 0.7565 |
| Comp. | 1.0000 | 0.9945 | 0.9767 | 0.9542 | 0.9291 | 0.8977 | 0 8652 | 0.8360 | 0.8015 | 0.7680 |
| △ | 0 | + 3 | − 11 | − 12 | + 25 | + 30 | + 2 | − 24 | − 71 | − 115 |
| λ 1.655 Obs. . | 1.0000 | 0.9965 | 0.9820 | 0.9657 | 0.9504 | 0.9275 | 0.9012 | 0.8770 | 0.8471 | 0.8155 |
| Comp. | 1.0000 | 0.9960 | 0.9831 | 0.9651 | 0.9486 | 0.9252 | 0.9012 | 0.8795 | 0.8529 | 0.8286 |
| △ | 0 | + 5 | − 11 | + 6 | + 18 | + 23 | 0 | − 25 | − 58 | − 131 |
| λ 2.097 Obs. . | 1.0000 | 0.9965 | 0.9858 | 0.9709 | 0.9563 | 0.9361 | 0.9149 | 0.8924 | 0.8664 | 0.8377 |
| Comp. | 1.0000 | 0.9966 | 0,9856 | 0.9702 | 0.9558 | 0.9356 | 0.9149 | 0.8963 | 0.8732 | 0.8522 |
| △ | 0 | − 1 | + 2 | + 7 | + 5 | + 5 | 0 | − 39 | − 68 | − 145 |
| $\frac{1 + 2 \cos i}{3}$ | 1.0000 | 0.9865 | 0.9443 | 0.8901 | 0.8399 | 0.7743 | 0.7101 | 0.6561 | 0.5946 | 0.5415 |

We see from table 3 that $a\lambda$ is nearly constant and not far from 2.0 throughout the photographic and visual parts of the spectrum. Further the residuals in table 4 must be considered rather small when we consider the magnitude of the observed temporary changes of $f(i)_{obs}$[1], and thus we must admit that the results in the two tables strongly support our main assumptions above.

There is, however, a perceptible decrease of $a\lambda$ with the wave-length, and a marked systematic run of the residuals in table 4. In constructing this table the separate values of $a$ in table 3 have been used, but from a certain point of view we are more interested in the residuals which we get by assuming $a\lambda$ to be constant, say = 2.0, for all wave-lengths. If we compare the new values $a = \frac{2.0}{\lambda}$ with the old ones in table 3, and look for corresponding wave-lengths in the first line of that table, we can find approximately the new values of $f(i)_{comp}$ by interpolation between two consecutive wave-lengths in table 4. It is evident at once that the residuals must still be very small for the shorter wave-lengths; in fact this will be the case also for the largest wave-lengths, in spite of the decrease of $a\lambda$, by reason of the small range of $f(i)$ for this part of the spectrum.

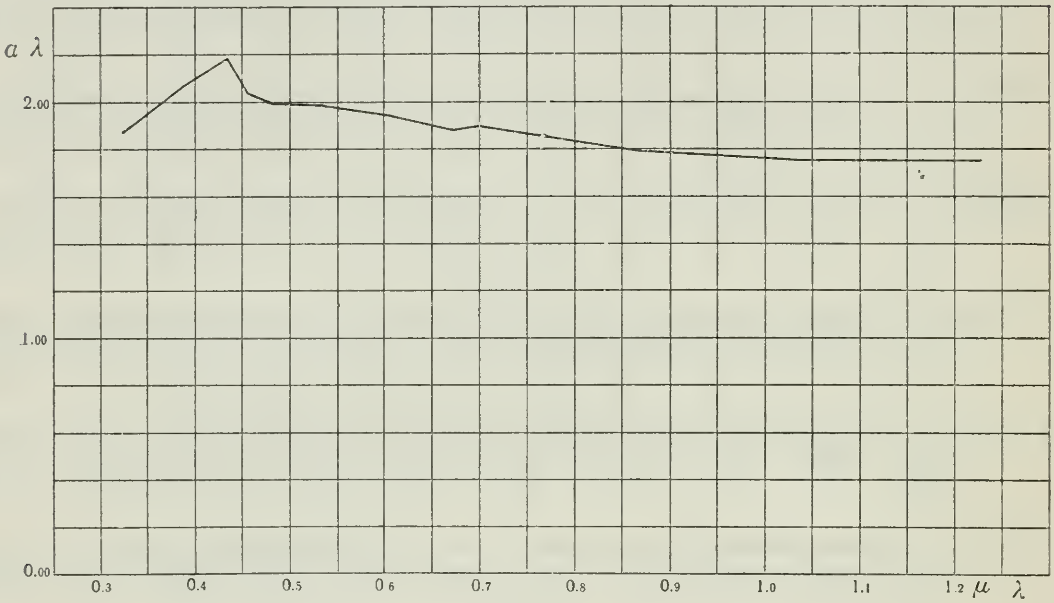

Fig. 2.

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[1] See Annals of the Astrophys. Obs. of the Smithsonian Inst. Vol. II Part. III; Vol. III. Ch. VII.

It may perhaps be worth while mentioning that the calculations leading to tables 3 and 4 were at first carried out on the supposition that $K_{\vartheta,\varepsilon}$ could be put equal to unity. Then the systematic change of $\alpha\lambda$ in table 3 was considerably greater; between wave-lengths $1.225\,\mu$ and $0.456\,\mu$ there was an increase from 1.37 to 1.94 instead of from 1.75 to 2.03. The systematic run of the residuals in table 4 was also somewhat more pronounced.

We may now explain a variation of $\alpha\lambda$ by introducing other values of $k$ in equation (10). We put

$$\alpha' = \frac{c}{\lambda\tau\sqrt[4]{1+2k}}\,, \qquad M' = \sqrt[4]{1+2k}\,(1+2k\ \cos\ i)^{-\frac{1}{4}}\,.$$

If $k$ is not far from unity we may assume the new values of $K$ and $K_o$ to be equal to the old ones. Then we have

$$\frac{e^{\alpha'}-1}{e^{\alpha' M'}-1} = \frac{e^{\alpha}-1}{e^{\alpha M}-1}\,,$$

and for short wave-lengths nearly

$$\alpha'(1-M') = \alpha(1-M)\,.$$

Now we assume $\alpha'\lambda\sqrt[4]{1+2k}$ to be constant for all wave-lengths and equal to $\alpha'_o\lambda_o\sqrt[4]{3}$, where $\lambda_o$ is the wave-length for which we have $k=1$. Then we get

$$M'-1 = (M-1)\,\frac{\alpha\lambda}{\alpha'_o\lambda_o}\left(\frac{1+2k}{3}\right)^{\frac{1}{4}} \quad . \quad . \quad . \quad . \quad (12)$$

For $\alpha'_o\lambda_o$ we assume the value 2.00; in the first approxima- we put $\left(\frac{1+2k}{3}\right)^{\frac{1}{4}} = 1$, then $M'$ may be derived, and $k$ by means of the formula

$$k = \frac{M'^4-1}{2\,(1-M'^4\ \cos\ i)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

In a following approximation we compute a better value for the last factor in (12), and derive a new value of $k$.

If $\alpha\lambda$ decreases, $k$ will decrease also; for $\alpha\lambda = 1.90$ we get $k = 0.85$. In this way $\varkappa + \sigma$ would increase with the wave-length from $\lambda$ 0.443 $\mu$ towards longer wave-lengths. But the systematic residuals for large values of $\varrho$ in table 4 would be increased by the new formula, and therefore it is perhaps not permissible to draw a conclusion in this direction.

On the other hand the correction to »Schuster's approximation», which is of greater importance for long wave-lengths because the assumption $k = 1$ cannot be far from the truth, tends to a somewhat smaller energy-fall than that given by our formula, and would thus tend to increase the values found for $a\lambda$. It is probable that the systematic decrease of $a\lambda$ towards long wave-lengths may be accounted for in this way.

The conditions are somewhat different for the three shortest wave-lengths in the tables. There is a sharp falling-off in $a\lambda$ from $\lambda$ 0.433 $\mu$; for this wave-length $a\lambda$ is abnormally large, for $\lambda$ 0.323 $\mu$ it is, on the contrary, remarkably small. In addition the residuals in table 4 for $\varrho = 0.92$ and 0.95 change sign when we go to the three shortest wave-lengths, the observed values being greater than those given in the formula; at the same time the residuals for small values of $\varrho$ have numerically large negative values. Now we must remember that the observed values for $\lambda$ 0.323 $\mu$ are taken from another source than those of the other wave-lengths; in addition the values for $\lambda$ 0.386 have perhaps not the same accuracy as the others. But probably this cannot explain the facts mentioned.

The decrease of $a\lambda$ for the wave-lengths 0.386 $\mu$ and 0.323 $\mu$, as well as the run of the residuals for these wave-lengths in table 4, agree well with a slight decrease of $k$, that is a real increase of $\varkappa + \sigma$, for the effective layers of the radiation. The most obvious explanation is that the scattering according to Rayleigh's law becomes perceptible for these wave-lengths in the extreme violet. For the wave-length 0.433 $\mu$ the conditions seem to be somewhat irregular; this may be connected with the fact that this wave-length falls in the immediate vicinity of the accumulation of absorption lines around the Fraunhofer line G. It is possible that line absorption has caused disturbancies in the run of $f(i)$ for the wave-lengths 0.386 $\mu$ and 0.323 $\mu$ as well. In fact, if the influence of the lines consists in changing slightly the radiation for the different values of $\varrho$ relative to the radiation from the center of the disk, the run of the residuals and the great value of $a\lambda$ for $\lambda$ 0.433 $\mu$ may be explained. But then the values of $a\lambda$ for

$\lambda$ 0.386 $\mu$ and $\lambda$ 0.323 $\mu$ in table 3 may also be too great; if the lines were absent we should perhaps find still smaller values of $k$, indicating a still greater importance of the opalescence scattering for these wave-lengths. But on the other hand there is probably a rather strong scattering in the ultra-violet lines which may contribute to the decrease of $k$; thus the general scattering may be of quite subordinate importance for all wave-lengths here considered. With these questions we are lead over to consider the nature of the Fraunhofer lines.

In the introduction it was mentioned that for the extreme violet region of the spectrum the line absorption seems to be of rather great importance for the average run of the energy-curve, and by this reason it would be a matter of the greatest importance to know more thoroughly the conditions that prevail in the process of absorption and emission for these singular wave-lengths. For the lines of hydrogen we know, for instance, that the radiation follows qualitatively the laws of temperature radiation, that the power of extinction is related to the power of luminiscence, and that with increasing strength of excitation the relative intensity of the short wave-lengths increases[1].

According to W. H. Julius an absorption line in the solar spectrum would be mainly formed by anomalous refraction and anomalous scattering. We leave out of consideration here the repulsion between the components of narrow pairs of lines which, according to Julius, ought to be a consequence of this theory, and the existence of which is in dispute[2]. But if the power of emission of the gases may be neglected, a further consequence would be, according to the results of Schwarzschild referred to above, that the intensity of radiation in a line ought to be diminished in the proportion 3 : 1 from the center of the solar disk to the limb. Because the intensity-fall towards the solar limb for the continuous spectrum increases considerably above this proportion in the violet region, we should expect the absorption lines of short wave-lengths to be less conspicuous at the limb than at the center, and the contrary for lines of longer wave-lengths. A decided difference of this kind between different spectral regions does not seem to be present. except perhaps for the strong ultra-violet lines. Further, from JULIUS' point of view the agreement between the line intensities in the spectra of giants and dwarfs of the same spectral type would be hard to understand, the former stars probably having considerably greater atmospheric masses. Therefore we may doubt it the nature of the extinction can really be characterized as a pure scattering.

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[1] See Kayser, Handbuch der Spektroskopie, Vol. V, p. 486.

[2] According to A. S. King (Astrophys. Journ. 45, p. 254, 1917; Mt Wilson Contr. No. 130) such an effect cannot be discovered for electric furnace lines showing strong anomalous dispersion.

From the similarity of the solar absorption spectrum at center and limb we may, in fact, draw some conclusions about the atmospheric line radiation. We denote by $s$ the coefficient of extinction for a line, then we may define a function $F$ so that the total emission per unit volume of a layer for the line in question is $4\pi s F$. The function $F$ is thus analogous to the function $E$ for a radiation of temperature. We may conclude that in the effective layers of a line $F$ must increase towards lower levels: if it did not, the intensity of radiation in a line would not decrease towards the limb. If we were concerned with a pure scattering of an incident radiation $A\,ds\,d\omega$ equally distributed over one hemisphere, we should have $F = \frac{1}{2} A$. Now, as before, we define the optical mass

$$d\mu' = s dx\,,$$

and, as we may suppose $F_0 > 0$, we put

$$F = F_0\left(1 + g(\mu') \cdot \mu'\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where $F_0$ and $g$ are functions of $\lambda$. If the lines were formed by pure scattering, we should have $g = 2$ for all wave-lengths.

For the continuous radiation we may put a similar expression

$$E = E_0\left(1 + f(\mu) \cdot \mu\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

$E_0$ and $f$ are variable with the wave-length; further, they are functions of the limiting temperature $\tau$.

The appearance of the lines at the solar limb would now be explainable, if we could suppose a common radiative equilibrium for lines belonging to the same element in analogy with the equilibrium for the continuous radiation. If we suppose for the different wave-lengths approximately the same deviations of $g(\mu)$ and $f(\mu)$ from the mean value $\overline{g} = \overline{f} = 2$, we should have, according to p. 36, nearly the same relative intensity-fall along the apparent solar radius for the lines as for the continuous spectrum.

Such a radiative equilibrium for the lines would mean that the energy absorbed in a certain spectral line is spread over and emitted by a certain number of different spectral lines, and in varying proportions for different states of excitation; the degree of excitation is measured by the value of $F$. Thus it seems as if we must assume a connection between the centra of resonance for different lines permitting an exchange of energy from one line to another; we may imagine that there are generally different processes of emission by which a single radiating atom can return into its original state before the absorption, but it is also possible that we are here concerned to some extent with an influence of collisions between different radiating atoms. When only the absorbing wave-length is left open for emission, the process must be a pure scattering of the radiation.

Now it seems from the appearance of giants and dwarfs of the same spectral type as if the transparency of the atmospheres for the greatest part of the lines were independent of the size of the atmospheres, and the question arises if this can be explained according to the principles mentioned above.

Let us consider the effective layers of a number of lines of a certain element which we suppose to form a system in radiative equilibrium. Now we know, especially from the investigations of Ch. E. St. John[1] not only that different elements have different effective layers in the solar atmosphere, but also that lines of the same element occur up to very different heights; thus the strong lines have, on an average, higher effective layers than the weak ones. The weakest lines in the solar spectrum are probably formed in levels with rather great atmospheric pressure. Then we may suppose that the members of the special system of lines considered here have different heights too, owing to special conditions of radiation at different levels. The reason why the solar gases absorb and emit any discontinuous radiation at all may be sought for in the high temperature of the gases, or perhaps in the case of some elements, for instance hydrogen, in the occurrence of electric discharges. In the state of radiative equilibrium the power of resonance for a certain line may depend on the general strength of excitation, but also on the temperature and density of the gaseous layers, or on the presence of vapors of certain other elements. It this way the system will change from one layer to another; the radiation of some lines will at certain points of the atmosphere pass over into layers, in which the general continuous radiation for those wave-lengths prevails. Then if we consider high and low level lines of the same system, and if we consider the upper limit of the effective layers for the latter, we find that the state of equilibrium demands a certain minimum intensity of the discontinuous radiation. For if the average continuous radiation towards lower levels for the wave-lengths of the low level lines were in a certain proportion more intense than the corresponding radiation for the high level lines, it would tend to increase the degree of excitation of the whole line system.

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[1] Ch. E. St. John, Astrophys. Journ. 38. p. 341, 1913; Mt. Wilson Contr. No. 74. Astrophys. Journ. 40, p. 356, 1914; Mt. Wilson Contr. No. 88.

At a large depth, in the photosphere itself, the discontinuous radiation must certainly be incorporated in the general radiation of heat; we assume that it gets the character of independent discontinuous radiation at a certain point of the radius in the thin gases of the solar atmosphere. We may imagine that the position of this point is determined principally by a certain critical value of the excitation, for it is probable that the atomic resonators cannot bear a too great excitation without leaving the regular emission of distinct spectral lines and joining the general system of temperature radiation. Further it is probable that, if there were no perceptible continuous radiation in the atmosphere. the point in question would be situated at a rather low level — making the lines appear very dark in the solar spectrum, because the coefficient of extinction is likely to increase rapidly in the dense and hot inner layers — but that a minimum value of $F_0$ is actually fixed by the continuous radiation inwards from the atmospheric envelope in the way suggested above, and that for this reason the low limit of the effective layers is shifted towards a higher level. It is possible that there may be a serious conflict betwen the conditions of radiation at the low and high limits of the effective layers, but on the other hand the controlling power of the steady flow of continuous radiation is probably very strong, being able to regulate during immense spaces of time perhaps even the relative frequency of different elements at different levels.

We shall try to examine the matter for a somewhat simplified system of lines. We assume the coefficient of extinction to be the same for both high and low level lines in their common effective layers; in the high atmospheric layers the high level lines are assumed to have the coefficient of extinction $s_1$ per unit mass of the atmospheric gas. The coefficient of extinction per unit mass for the continuous radiation in the same layers is taken to be $\varkappa_1$. We assume both sorts of lines to be fairly uniformly distributed over the spectrum and consider the radiation inwards at the average upper limit of the effective layers of the low level lines. Suppose m is the atmospheric mass between the upper limits of the effective layers for the two sorts of lines and that the high level lines attain a very great height. Then, according to equations (6), the radiation inwards for the high level lines is $s_1 \mathrm{m} \cdot 2\overline{F}_0$, and the continuous radiation of the wave-lengths corresponding to the low level lines is $\varkappa_1 \mathrm{m} \cdot 2\overline{E}_0$; $\overline{F}_0$ and $\overline{E}_0$ are average values of $F_0$ and $E_0$ from different spectral regions. Equating the two expressions, we get

$$\overline{F}_0 = \frac{\varkappa_1}{s_1} \overline{E}_0 ; \quad \ldots \ldots \ldots \quad (16)$$

the average intensity for a high level line relative to the surrounding spectrum will then be $\frac{\varkappa_1}{s_1}$, independently of the atmospheric mass. For the low level lines the intensity in a line will be greater, because we look through more transparent optical matter into layers of higher excitation. For this reason the low level absorption lines of a system will fade in the solar spectrum relative to the high level ones. Of course all lines will fade when their effective layers are placed near the photosphere and the continuous radiation inwards and outwards from the overlying vapors becomes relatively strong.

For a giant of solar type we may expect a considerably larger atmospheric mass per unit surface of the photosphere than for the sun; but according to the arguments above the general appearance of the spectral lines ought still to be the same. But if the power of absorption for a low level line decreases rapidly with decreasing density, which ought to be the case especially for such low temperature lines as appear faint in stellar spectra (other low temperature lines ought to be dark, their effective layers extending into high levels), the mass m defined above must be larger for the giant, because the effective layers of the low level line in question will be more or less strongly confined to the vicinity of the photosphere. The intensity of radiation in the effective layers of the low level line must therefore be larger in the giant, and consequently the resulting extinction will be diminished. The corresponding absorption line ought thus to fade in the giant spectrum.

On the other hand, in the case of those lines the absorptive power of which demands a certain minimum degree of excitation the mass m ought to be the same for the dwarf and the giant; in the latter star the effective layers will thus have smaller pressures and densities than in the former. For this reason we must expect a change to decidedly stronger coefficients of extinction in the effective layers of enhanced low pressure lines[1], as we pass from the dwarf to the giant. From these conclusions it seems as if we may explain from our point of view the changes of line intensities with stellar luminosity on which ADAMS and KOHLSCHÜTTER's method depends[2].

The strongest lines observed by W. S. ADAMS[3] in the solar spectrum seem to show a somewhat greater contrast to the continuous spectrum at the limb than at the center. Then, according to our theory, in the case of the low level lines of a system the contrast to the continuous spectrum ought to be more constant over the disk and ought ultimately to decrease towards the limb, because for those lines we look more directly into layers where $F$ is greater and therefore the relative increase $\frac{\Delta F}{F}$ per unit optical mass becomes smaller. The observations show that the faint lines of very heavy elements, as well as the enhanced lines with great intensity in the flash spectrum, fade strongly towards the limb. This is also the case with the wings shown by many of the strong lines and it indicates that they are formed at relatively low levels. It seems as if — in accordance with our theory — there were the general rule that lines with great excitation even in their upper effective layers fade at the limb; those of weak excitation are more constant or even strengthened.

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[1] That the enhanced lines *Ti* are favoured by low pressures has been found by H. G. GALE and W. S. ADAMS, Astrophys. Journ. 35, 10, 1912; Mt. Wilson Contr. No. 58.

[2] See W. S. ADAMS and A. H. JOY, loc. cit.

[3] W. S. ADAMS, Astrophys. Journ: 31, p. 30; Mt. Wilson Contr. No. 43.

We must now consider the possibility that from a certain level a high level line, or a couple of such lines in a certain spectral region, may be left as the only members of a system. Then we shall have for such lines a pure scattering with $g = 2$, and the actual value of $F_0$ for these lines will depend on the scattering mass. These facts will be of importance for the appearance of the flash spectrum, because the magnitude of $F$ will be decisive for the visibility of flash lines of the same effective level in the solar atmosphere. For at the solar limb we look through gaseous matter of very great extension in the line of sight and therefore the magnitude of the absorptive power will be of subordinate importance.

When the heights of the lines in the flash spectrum photographed by Mitchell were put in relation to the wave-length, St. John[1] found a pronounced increase of height from the long wave-lengths into the ultra-violet part of the spectrum; this fact may be due to an increase in the height of the effective layers or to a regular increase of $F_0$ with the wave-length. The latter explanation is, in fact, for relatively long wave-lengths a natural consequence of our theory. But according to (16) we must expect a decrease of $F_0$ when we go to the ultra-violet lines; this conclusion is confirmed by the fact that the difference betwen flash and solar intensity decreases very strongly when we pass from the violet to the ultra-violet region[2]. The decrease of intensity outwards at the limb itself is very strong in the latter region; according to Schwarzschild[3] the ultra-violet radiation is quite predominant at the extreme strip of the solar disk (breadth of the strip only some sec. of arc). The visibility of ultra-violet flash lines up to very high levels seems thus to indicate abnormally high values of $F$ in the upper atmospheric regions, and it seems as if we may conclude that the character of the extinction in the ultra-violet lines is here successively transformed into a pure scattering.

We must add that the intensities at the solar limb of strong ultra-violet absorption lines given by Adams support this view. From 40 lines between $\lambda$ 3741 Å and 3910 Å we find by adding together the intensities for center and limb on the Rowland scale that the ratio of intensity center: limb is 1.48. For the region λ 5000—5500 Å we have from 60 lines the proportion 0.98. In this region two lines with intensities exceeding 20 were excluded in order to make the two regions more comparable. Further, 50 lines of corresponding intensities between λ 4200 Å and 4400 Å gave the value 1.09 for the absorption line intensity at the center relative to that at the limb. This seems to establish the difference between the strong violet lines and the strong ultra-violet ones; the latter seem to form more or less separate systems.

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[1] Astrophys. Journ. 40, p. 372; Mt. Wilson Contr. No. 88.

[2] loc. cit.

[3] K. Schwarzschild Abh. der K. Ges, der Wiss. zu Göttingen, N. F. Bd. V, No. 2, 1906.

A matter of the greatest importance in this connection is that we have reason to think that the transparency of the solar envelope for the high level ultra-violet lines cannot be regulated in quite the same way as was probable for other lines. According to Planck's theory of energy-quanta the absorption and emission in the radiating atom is a process of greater exchange of energy for short wave-lengts than for long ones, the energy-quantum being inversely proportional to the wave-length, and at least if we try to connect our theory with Bohr's[1] model of the radiating atom, we must conclude that a distribution of the energy absorbed in a line over a series of lines ought to occur mainly from lines of short wave-length to lines of longer wave-lengths, in accordance with the laws of fluorescence phenomena. Accordingly the low level lines preserving the transparency of the solar envelope from decreasing ought on an average to be placed to the violet side of the high level lines, in order to be effective in this respect. In the ultra-violet region, however, the continuous radiation inwards from the atmospheric layers must be very small, probably too small to regulate perfectly, in spite of occasional disturbancies of the equilibrium, the intensity of radiation in eventually coordinated high level lines. Therefore we have some reason to believe that the strong ultra-violet lines of high levels form more or less separate systems, and that the resulting extinction is not controlled by the continuous radiation of the atmospheric envelope but increases with increasing atmospheric mass. This view is supported to some extent by the strong development of ultra-violet lines in stellar spectra of late types. Just because of the accumulation of strong absorption lines in the ultra-violet region the effect of a probable increase of atmospheric mass from a dwarf to a giant ought to be very perceptible in this part of the spectrum. It is of great importance in this connection to note that the selective power of extinction seems to be distributed over considerable parts of the continuous spectrum, the lines being involved in strong »wings»; the large amount of anomalous dispersion found by A. S. King[1] for the strong ultra-violet lines of iron, chromium and titanium is of interest here. For stars of very strong luminosity the lines and their wings ought almost to »cut off» the extreme part of the continuous spectrum.

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[1] See for inst. N. Bohr. Phil. Mag. 26, p. 1, 1913.

## 3. The solar energy-curve.

The discussion of all the matters treated above becomes very important, when we pass on to the ultimate object of this chapter, the solar energy-curve. In agreement with the statements on page 38, we see from table 4 that $\lambda$ 0.501 $\mu$ is very near to the medium wave-length for which the energy-distribution over the solar disk follows most closely the approximate expression for a radiative equilibrium, $F(i) = \frac{1 + 2 \cos i}{3}$, tabulated in the last line of the table. The assumption $k = 1$ must be most strictly applicable to this wave-length, and thus we have some reasons for regarding the value of $\alpha\lambda$ for this region of the spectrum as especially characteristic for the actual conditions of temperature in the solar envelope. Now, according to table 3, we have $\alpha\lambda$ very nearly equal to 2.00 from $\lambda$ 0.456 $\mu$ to $\lambda$ 0.534 $\mu$. This value is moreover almost valid for all values of $\varrho$ less than 0.825; the residuals for $\varrho = 0.92$ and 0.95 would demand $\alpha\lambda$ to be somewhat larger. As we may expect the greatest deviations from our theory, on account of the necessary simplifications, for great values of $\lambda$ and $\varrho$, we thus adopt, remembering the signification of $\alpha\lambda$,

$$\frac{c}{T_0} = 2.00\,,$$

where $T_0$ is the effective temperature of the sun's center as defined above. For $c$ we adopt the value[1] 14300, and thus we have

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[1] A. S. King, Astrophys. Journ. 45, p. 254, 1917; Mt. Wilson Contr. No. 130.

$$T_0 = 7150^0$$

For the temperature of the extreme atmospheric layers we get

$$\tau = 5430^0 .$$

The effective temperature of each wave-length for the total radiation of the solar disk will be

$$T = \frac{T_0}{\overline{M}},$$

where $\overline{M}$ is the value of $M$ corresponding to the zone of the disk where the radiation from unit surface is equal to the mean surface brightness for the whole disk. If every point of the solar disk were made to have this particular value of $M$, that is the effective temperature $T$, and the corresponding value of $i$ in $K_{\varsigma,\varepsilon}$, then the total radiation of a certain wave-length reaching the earth would still be the same as before. The mean surface brightness for each wave-length was determined by dividing the solar disk into ten zones, and multiplying the area of each zone by the corresponding intensity from the table of ABBOT, FOWLE and ALDRICH (the observed values in table 4); if the zonal areas are expressed in the area of the whole disk as a unit, we have at once the average intensity. The particular zone having this average intensity was then found by graphical construction. The radius in question is nearly the same for all wave-lengths; we find $\varrho = 0.74$ and thus

$$\log \overline{M} = 0.02675 ,$$

and finally

$$\boldsymbol{T = 6720^0}$$

Thus the solar energy-curve will be given by the formula

$$I = K_{\theta,\varepsilon}\, C_0 \lambda^{-5} \left( e^{\frac{c}{\lambda T}} - 1 \right)^{-1} , \quad . \quad . \quad . \quad . \quad . \quad (17)$$

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[1] Tätigkeitsbericht der Phys.-Techn. Reichsanstalt, Zeitschrift für Instrumentenkunde, Bd. 36, p. 85, 1916.

where we have to put

$$T = 6720^0 ,$$

and further

$$\theta = \frac{c}{\lambda T}, \quad \varepsilon = 2 \cos \bar{i} = 1.345 ,$$

where $\bar{i}$ is the angle $i$ corresponding to $\varrho = 0.74$.

If we want $I$ to give the intensity of the solar radiation at the distance of the earth, measured in gr. calories per cm$^2$ per minute, we derive $C_0$ in the following way. We have $C_1 = \pi C$ for the constant in Planck's formula applied to the total radiation outwards from a black body surface. Now we have for the constant $C_1$[1]

$$C_I = \frac{c^4 \sigma}{6 \cdot 1.0823},$$

where $\sigma$ is the constant in Stefan-Boltzmann's law. If the intensity of radiation is measured in the units cited, we have, according to Kurlbaum's[2] determination,

$$\sigma = 76.8 \cdot 10^{-12} ,$$

and thus, $c$ being assumed to be 14300,

$$\log C_1 = 5.69422 .$$

If $r$ is the radius of the sun, $R$ the distance between the sun and the earth, we have

$$C_0 = \frac{r^2}{R^2} C_1 ,$$

and thus, as $\frac{r}{R}$ is very nearly = sin 16′.0, we get

$$\log C_0 = 1.02991$$

For the energy-curve observed I have taken the values of C. G. Abbot[3], which are derived as the mean of several determinations at different times and with different instruments at the three stations of Washington, Mount Wilson, and Mount Whitney. The values for the separate wave-lengths, which are given on an arbitrary scale, were reduced in such a way that the surface below the curve should be equal to the average value of the solar constant 1.932, given by Abbot, Fowle and Aldrich. By graphical integration the surface was found to be 3.352 on the arbitrary scale. The factor $f$ with which the observed values are to be multiplied will thus be

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[1] See for inst. M. Planck, Ann. der Physik. 4, p. 562, 1901.

[2] F. Kurlbaum, Wied. Ann. 65, p. 759, 1898. More recent investigations indicate a value of $\sigma$ a few per cent higher.

[3] C. G. Abbot, Astrophys. Journ. 34, p. 197, 1911; Annals of the Smithson. Inst., Appendix to Vol. III.

$$f = \frac{1.932}{3.352},$$

$$\log f = 9.76071 .$$

For the wave-length 0.6 $\mu$ we get, for instance, by applying the formula (17) the intensity $I_{comp} = 3.933$. The observed value, reduced in the manner described, will be $I_{obs} = 2.906$. If we use the notation $\gamma = \frac{I_{obs}}{I_{comp}}$, we get

$$\gamma = 0.739$$

We may account for the factor $\gamma$ by changing $T, c$, or $\sigma$. We must, however, consider the coincidence betwen $I_{comp}$ and $I_{obs}$ to be rather close, because $\gamma$ is very sensitive to small changes in $T$ and $c$. In order to get $\gamma = 1$ we should have $T$ about 6200$^0$, or $c = 13300$, but we prefer to keep the old values unchanged and multiply our values of $I_{comp}$ with the value of $\gamma$ found, in order to get coincidence at the wave-length 0.6 $\mu$. The new intensities thus derived are denoted by $I'_{comp}$.

The observed and computed intensities, reduced in the ways described above, are now tabulated in table 5. The last line of the table contains the factor $K_{\vartheta,\varepsilon}$.

Table 5.

| $\lambda$ | 0.30 $\mu$ | 0.35 $\mu$ | 0.40 $\mu$ | 0.45 $\mu$ | 0.47 $\mu$ | 0.60 $\mu$ | 0.80 $\mu$ | 1.00 $\mu$ | 1.30 $\mu$ | 1.60 $\mu$ | 2.00 $\mu$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $I_{obs}$ | 0.311 | 1.547 | 2.500 | 3.474 | 3.597 | 2.906 | 1.536 | 0.955 | 0.518 | 0.307 | 0.142 |
| $I'_{comp}$ | 3.094 | 3.633 | 3.853 | 3.791 | 3.714 | 2.906 | 1.736 | 1.022 | 0.492 | 0.261 | 0.126 |
| $K$ | 1.139 | 1.048 | 1.011 | 0.989 | 0.983 | 0.961 | 0.954 | 0.954 | 0.957 | 0.960 | 0.963 |

The results are illustrated in fig. 3. The dotted curve is the energy-curve observed, the full-drawn one is the computed one. We ignore for a moment the behavior of the curves in the extreme violet, and state at once that the coincidence of the two curves between $\lambda$ 0.47 and $\lambda$ 2.00 $\mu$ is rather close. In this respect the effective temperature derived for the part of the spectrum about $\lambda$ 0.5 $\mu$ is thus nearly valid for longer wave-lengths too, even far beyond the visible spectrum. The inclinations of the two curves differ perceptibly for the wave-lengths 0.7 $\mu$ — 1.5 $\mu$; if we determine the effective temperature from the distribution of intensity in this region, we find a far too low value. At least to some extent this may be due to the correction which ought to be applied to »Schuster's approximation». On the other hand, for the wave-lengths 0.52 — 0.65 $\mu$ the strong inclinations of the two curves are almost exactly the same. It is interesting to compare this fact with the run of the residuals in the vertical columns of large $\varrho$ in table 4. There is a marked minimum of the residuals

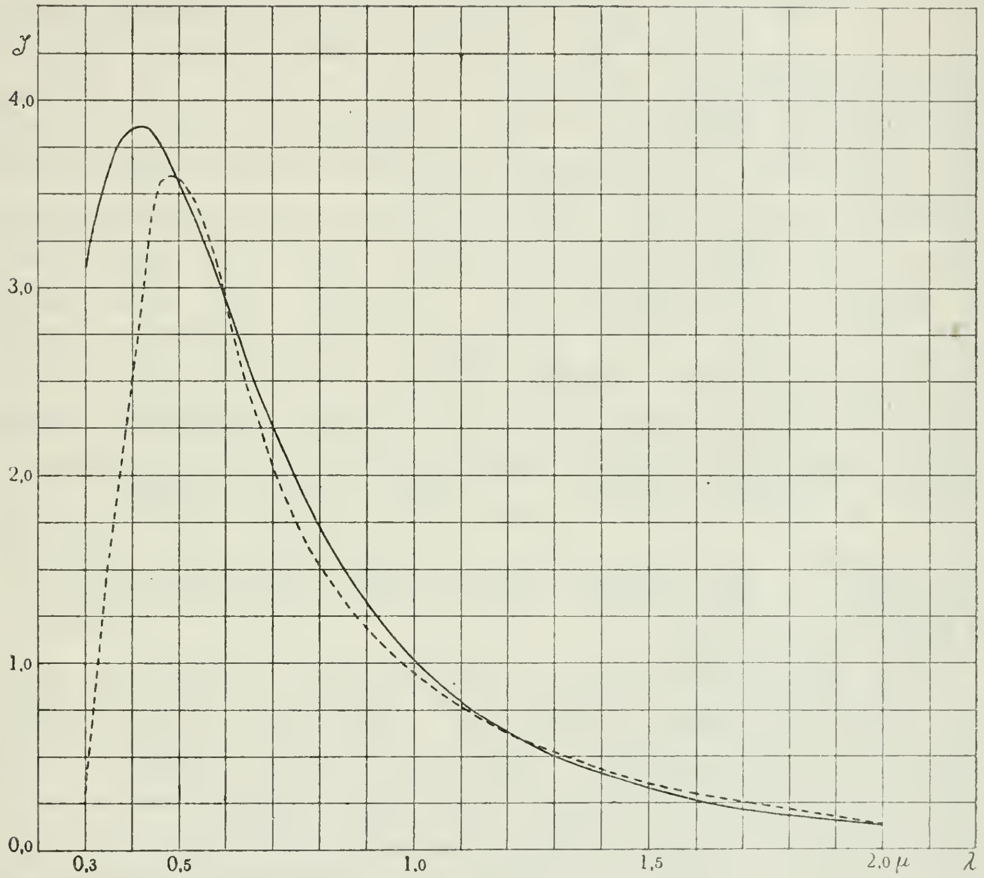

Fig. 3.

for $\lambda$ 0.534 $\mu$ and $\lambda$ 0.604 $\mu$; perhaps the coincidence of these two facts for this part of the spectrum is not a mere chance. Beyond $\lambda$ 2.00 $\mu$ the energy-curve observed declines rapidly, but the accuracy of the curve beyond this wave-length is rendered uncertain by the effect of terrestrial water vapor.

Previously we have seen that the introduction of the variable factor $K$ has diminished the change of the product $a\lambda$ with the wave-length as well as the magnitude of the residuals i table 4. Now the factor $K$ is a direct expression of the fact that the energy-curve is a blending of the radiation from successive layers of different temperatures, the layers being assumed to be arranged into radiative equilibrium. This equilibrium is, however, almost the same as that of the polytrope $n = 5$[1], that is a state not far from the isothermic one. The vertical movements which perhaps occur to some extent also in the outer layers of the solar vapors, for instance in granulations and sun-spots, must have a slight tendency to bring about an adiabatic equilibrium with a strong temperature gradient. Possibly some trace of discrepancy may be due to such perturbations of the radiative equlibrium. They would undoubtedly cause a more rapid rise of the maximum of energy on the side of the longer wave-lengths; further an adiabatic equilibrium leaves a strong intensity-fall on the solar disk; a tendency in this direction might explain the negative residuals of great $\varrho$ in table 4.

The phenomena related to the variability of the solar radiation found by Abbot, Fowle and Aldrich are of extraordinary interest in this connection. They find[2] that the increase of solar activity shown in an increase of the solar constant seems to run parallel with an increase of the number of sun-spots, a more rapid rise of the maximum of energy in the spectrum, and finally an increased falling off of the intensity along the apparent radius. The changes of contrast between center and limb are considerably larger than our residuals for the observed mean values i table 4. The maxima of the solar activity perhaps involve a disturbancy of the radiative equilibrium and an advance towards an adiabatic state.

The most conspicuous feature of the energy-curve observed is, however, the sudden decrease of intensity at $\lambda$ 0.45 $\mu$. We may state at once that a decrease of the constant $k$ made probable by the tables 3 and 4 and discussed on pp. 47, 48 would have the effect of diminishing the effective temperature $T$ in equation (17) for the violet region, and it is evident that the depression of the observed energy-curve may be partly due to such a cause, originating in the very effective layers of the radiation. For $\lambda$ 0.35 $\mu$ we may well reckon with a value of $k$ as small as 0.8, and this would cause a depression of the intensity for that wave-length in the proportion 1 : 0.83; the ratio between $I'_{comp}$ and $I_{obs}$ in table 5 is 1 : 0.43. It would be hard to reduce $k$ to a value which would correspond to the latter ratio.

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[1] R. Emden, Gaskugeln, p. 323; Teubner, 1907.

[2] loc. cit.

But there is also another circumstance of importance here. We have found above that the principle of radiative equilibrium leads on the whole apparently to a better concordance with the observations for the separate wave-lengths than for the integral radiation, as far as the distribution of intensity over the solar disk is concerned. We can understand the cause of this, if we notice the way in which the observed values of that table were derived. If the energy-curve did not show the depression in the extreme violet, the rapid energy-fall for these wave-lengths along the radius would have more influence upon the integral intensities for the different zones, leading to a better agreement with the formula.

This undoubtedly points to the conclusion that a part of the deformation of the curve takes place where the general value of $\varkappa + \sigma$ is already rather small, i. e. in relatively thin and high levels. In this way, though a scattering of opalescence, that is a general increase of $\varkappa + \sigma$, may have some influence, we must consider a great part of the depression to be due to an extinction in the very atmosphere of the sun, and probably to the influence of the strong accumulation of Fraunhofer lines in the extreme violet. If we suppose the lines to be produced by scattering with the coefficient $\sigma_0$, we should have

$$\mu_0 = \int_{-\infty}^{H_0} \sigma_0 dx$$

for the effective optical mass of the layers contributing sensibly to the formation of the line in question. At all events we assume that the depression of the energy-curve follows the law

$$R = \frac{S}{1 + \mu_0},$$

where $S$ is the original, $R$ the remaining intensity, $\mu_0$ a function of $\lambda$.

We now try to adopt for $\mu_0$ an emperical function, putting

$$\mu_0 = c_0 \cdot \lambda^{-n} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (18)$$

From the ratios of $I_{obs}$ and $I'_{comp}$ for wave-lengths $0.3\,\mu$—$0.4\,\mu$ in table 5 we find

$$n = 10,\; c_0 = 5 \cdot 10^{-5}.$$

We now use the notation

$$I''_{comp} = \frac{1}{1+\mu_0} I'_{comp}\,, \quad . \; . \; . \; . \; . \; . \; . \quad (19)$$

and for $\mu_0 = \frac{5 \cdot 10^{-5}}{\lambda^{10}}$ get the values of $I''$ in table 6.

Table 6.

| $\lambda$ | $0.30\,\mu$ | $0.35\,\mu$ | $0.40\,\mu$ | $0.45\,\mu$ | $0.47\,\mu$ | $0.60\,\mu$ |
|---|---|---|---|---|---|---|
| $I_{obs}$ | 0.311 | 1.547 | 2.500 | 3.474 | 3.597 | 2.906 |
| $I''_{comp}$ | 0.327 | 1.292 | 2.609 | 3.306 | 3.392 | 2.882 |

The above expression for $I''$ is only valid for the avarage intensity-fall towards the short wave-lengths. The real course of the energy-curve seems to have very rapid changes, and thus the rise of the curve from a region of specially strong selective extinction may certainly be much more rapid than that which follows the tenth power of the reciprocal wave-length.

Before we leave the discussion of the solar radiation, we shall briefly mention some values of the solar effective temperature, derived by different methods. These methods assume that a certain quality of the solar radiation may be represented by the corresponding property of a perfect radiator of a certain temperature. But because there are appreciable systematic deviations of the energy-curve from that of a black body, the different methods cannot give coincident results.

Stefan-Boltzmann's law applied to the value of the solar constant 1.932 gives the temperature 5830°, but it is evident that the law in question cannot give a value characteristic for the general energy-distribution in the spectrum, since a great part of the radiation in the extreme violet is extinguished, by which a considerable distortion of the energy-curve occurs. Nevertheless there is of course a special sense in which this value may be called the effective temperature of the sun; we then define this quantity as the temperature of a black body emitting the same integral energy per unit surface as the sun. In the foregoing we have found the temperature corresponding to the absolute intensities in the visible and infra-red spectrum according to Planck's law to be about 6200°, a value which is, however, subject to considerable uncertainty.

Wien's displacement law for the wave-length of maximum energy cannot be strictly applied, because the position of the maximum is probably altered by the extinction in the violet.

Another method employing the relative distribution of intensity between the wave-lengths of a certain spectral region depends on the solution of some equations formed in accordance with Planck's formula. For the energy-curve of Abbot this method leads to very different values for different parts of the spectrum[1]. Wilsing[2] applies this method to the determinations of the energy-curve derived by himself; between $\lambda$ 0.400 $\mu$ and $\lambda$ 2.128 $\mu$ he gets the value 5900° for the total radiation from all parts of the solar disk, 6900° for the radiation af the center. These values ought to be diminished[3] by about 120° to correspond to the value of $c = 14300$. These values seem, however, to be appreciably affected by the falling off of the energy-curve between $\lambda$ 0.420 $\mu$ and $\lambda$ 0.400 $\mu$. Now Wilsing applies the same method to Abbot's curves for different zones of the solar disk, excluding the wave-lengths 0.323 $\mu$ and 0.386 $\mu$. For the center he gets the temperature 6810° ($c = 14300$), for the limb he finds by extrapolation 5440°. According to Wilsing's theory the latter value ought to be the temperature of the surface layer of the cloudy photosphere. There is a very remarkable coincidence between this value of Wilsing and the value for the corresponding quantity $\tau$ in this paper, which was found to be 5430° by an examination of the energy-distribution along the radius of the solar disk.

In the theory of radiative equilibrium it is especially the quantity $\tau$ that has a real physical significance, the effective temperature for the center $T_0$ being in some degree a fictitious quantity, as long as we do not try to find where the effective layer corresponding to this temperature is situated. But as $K$ is very near unity, the meaning of the term effective temperature will be nearly the same as in the previous method cited. Our value for the total radiation of the disk, giving a good approximation to the intensity distribution of the visible and infra-red parts of the spectrum, is 6720°. The derivation of this value depends entirely on the value of $\tau$ and thus on the distribution of intensity along the apparent radius of the sun.

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[1] C. G. Abbot, The Sun, p. 112.

[2] Publ. des Astrophys. Obs. zu Potsdam, Nr. 72, 1917.

[3] loc. cit. p. 96.

# II. The influence of luminosity on the spectral energy distribution for late type stars.

For a discussion of the influence of varying luminosity on the distribution of spectral energy we start from the results for the solar radiation derived in the previous chapter. The sun is among the stars a dwarf of type $G_0$ in the Harvard classification; below we shall let it represent a dwarf of an arbitrary type not very different from the solar stage.

Equations (17) and (19) may now be condensed in the following way, the factor $K$ being put equal to unity,

$$I = \frac{C_0}{1 + \mu_0} \lambda^{-5} \left( e^{\frac{c}{\lambda T}} - 1 \right)^{-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (20)$$

where $T = \omega \tau$, $\tau = t\,(1 + 2m)^{-\frac{1}{4}}$, $m = \int\limits_{-\infty}^{H} (\varkappa + \sigma)\, dx$ Here $\omega$ is a constant equal for all stars, $\tau$ is the temperature of the extreme radiating layers, $t$ and $m$ are the temperature and the overlying optical mass for a layer of depth $H$ below a certain surface of reference. For the sun we had found $\tau = 5430^0$ and $T = 6720^0$, further $\mu_0$ on an average equal to $5 \cdot 10^{-5}\, \lambda^{-10}$.

Now for a solar star of arbitrary absolute magnitude we may put

$$T' = \omega \tau', \ \tau' = t'\,(1 + 2m')^{-\frac{1}{4}},$$

and we then have the ratio of the spectral intensities of the star and the sun

$$\frac{I'}{I} = \frac{e^{\frac{c}{\lambda T}} - 1}{c^{\frac{c}{\lambda T'}} - 1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (21)$$

where

$$\frac{T'}{T} = \frac{t'}{t} \left( \frac{1 + 2m}{1 + 2m'} \right)^{\frac{1}{4}},$$

for a spectral region where the intensities follow the laws of black body radiation; for another region we suppose

$$\frac{I'}{I} = \frac{1 + \mu_0}{1 + \mu'_0}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Now suppose we compare a giant and a dwarf of the same spectral type. Then it is very probable that for both stars we shall have to count the photosphere, i. e. the layer immediately below the effective layers of the absorption lines of the low level elements[1], from a point of the radius where the density of the vapors gets a certain value. We must suppose nearly the same temperatures for these critical layers; we then have the same pressures also, and the similarity of the absorption spectra, the characters of which depend on the partial density of the vapors of different elements, must seem quite natural.

W. S. Adams[2] has made an investigation of the differences of displacement for a great number of lines of different kinds in the spectra of Sirius, Procyon, and Arcturus relative to the solar spectrum, in order to get some knowledge about the effective pressures in the atmospheres of these stars. From the differences of displacement between enhanced lines and arc lines Adams finds for Sirius, spectral type A 0, and Procyon, a dwarf of type F 5, pressures 12 and 7 atm. greater than in the sun's reversing layer. For Arcturus, a giant of type K 0, the difference of displacement seems to be vanishingly small. Thus we must assume the atmospheric pressures to decrease towards later types; equality of the pressures in the reversing layers of giants and dwarfs is not at variance with these results.

1. According to the results of the researches in stellar spectrophotometry mentioned in the introduction we know that for the visual part of the spectrum the energy-curves of giant spectra follow nearly the laws of black body radiation. For the moment we therefore leave out the factor $\frac{1 + \mu_0}{1 + \mu'_0}$, which evidently becomes appreciable first in the violet region, and consider, to begin with, the general

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[1] It is evident that this definition of the photosphere is not the same as that given by J. H. Jeans, Monthly Notices 78, p. 35.

[2] W. S. Adams, Astrophys. Journ. 33, p. 64, 1911; Mt. Wilson Contr. No. 50.

luminosity effect included in the ratio $\frac{T'}{T}$ of the effective temperatures.

Suppose $m_0$ and $m'_0$ are the optical masses above the photospheres for the dwarf and the giant. For the atmospheric layers we may suppose $\overline{\varkappa}$ and $\overline{\sigma}$ simply proportional to the density, and we then have

$$\frac{m_0'}{m_0} = \frac{\mathrm{m}_0'}{\mathrm{m}_0},$$

where $\mathrm{m}_0$ and $\mathrm{m}_0'$ are the atmospheric masses. If M is the mass, $\overline{\varrho}$ the mean density, and $r$ the radius of the dwarf, and if we put $p_0$ for the pressure at the photosphere, we have, because the overlying shell of vapors may be regarded as being very thin,

$$p_0 = k\frac{\mathrm{M}\mathrm{m}_0}{r^2},$$

where $k$ is a constant, and

$$\overline{\varrho} = \frac{\mathrm{M}}{\frac{4}{3}\pi r^3}.$$

In the expression for $p_0$ we have not taken into account any attractive or repulsive force except gravitation. It is possible that radiation pressure may be of importance for the photospheric layers, at least from WILSING's point of view; for the purely gaseous atmospheric layers it is probably negligible.

Eliminating $r$ we have

$$\mathrm{m}_0 = C\cdot\frac{p_0}{\mathrm{M}^{\frac{1}{3}}\,\overline{\varrho}^{\frac{2}{3}}},$$

where $C$ is a numerical constant. Now we have a corresponding relation for the giant, and thus

$$\frac{\mathrm{m}_0'}{\mathrm{m}_0} = \frac{p_0'}{p_0}\left(\frac{\mathrm{M}}{\mathrm{M}'}\right)^{\frac{1}{3}}\left(\frac{\overline{\varrho}}{\overline{\varrho}'}\right)^{\frac{2}{3}} \quad . \; . \; . \; . \; . \; . \; . \quad (23)$$

Now we must turn to an estimate of the ratios $\frac{\mathrm{M}}{\mathrm{M}'}$ and $\frac{\overline{\varrho}}{\overline{\varrho}'}$. Here we may get valuable knowledge from an investigation of

H. N. Russell's[1], in which this author has related the most important facts leading to a division of the late type stars into giants and dwarfs. From direct measurements of stellar parallaxes Russell computes the mean absolute magnitudes of giants and dwarfs of different spectral types; from about 550 visual pairs showing a trace of relative motion he derives the mean absolute magnitudes of the same types reduced to the solar mass, and from 80 eclipsing binaries he gets the densities of giant stars, i. e. their absolute magnitudes reduced to both the surface brightness and the mass of the sun.

The first two sources give the mean masses of different stellar types; Russell gets the results in the following table.

Table 7.

| Spectrum | Average mass | |
|---|---|---|
| | Giants | Dwarfs |
| B 2. . . . | 3.0 | — |
| A 0. . . . | 2.3 | — |
| A 5. . . . | — | 1.2 |
| F 0. . . . | — | 1.7 |
| F 3. . . . | — | 0.8 |
| F 8-G 0 . | — | 0.5 |
| G-G 5 . . | 3.0 | — |
| G 5. . . . | — | 0.3 |
| K 0. . . . | 1.5 | 0.3 |
| K 5-M . . | 1.5 | 0.2 |

These results are derived for the bright components of double stars; for our purpose we let them represent the actual conditions of the types in the general stellar system, and thus have for the ratio of the masses dwarf: giant approximately

$$\frac{M}{M'} = \frac{1}{7}$$

From the two last sources above we get the mean surface brightness of the giants; the results agree well with the observed color-indices. From the mean absolute brightness per solar mass, and with the known surface brightness relative to the sun, we may now estimate the mean densities for different types. For the dwarfs the density may be considered constant, the change in absolute magnitude being accounted for by the decrease of surface brightness; for the giants Russell obtains for the densities relative to that of the sun the following values, agreeing well with the values from the eclipsing binaries,

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[1] H. N. Russell, Popular Astronomy 22, pp. 275, 331; 1914.

$$\text{Type G}, \frac{1}{350}; \text{ K}, \frac{1}{2800}; \text{ M}, \frac{1}{25000},$$

When we consider stars of the same spectral type, we make the supposition $p_0 = p_0'$ in agreement with the preceding discussion, and then derive the following values of $\frac{m_0'}{m_0}$ according to equation (23)

$$\text{Type G}, \frac{m_0'}{m_0} = 26; \text{ K}, \frac{m_0'}{m_0} = 104; \text{ M}, \frac{m_0'}{m_0} = 855,$$

Now we may turn back to equation (21). According to the facts just mentioned about the surface intensities of giants and dwarfs it is obvious that $\frac{T'}{T}$ cannot differ very much from unity. In the introduction it was mentioned that the change found for the visual color of Osthoff per unit absolute magnitude for the types F 5-K almost corresponds to the change of the effective wave-length, valid for the photographic part of the spectrum. This effect, which persists nearly unchanged through a considerable part of the spectrum, must be thought to be due mainly to a change of temperature according to equation (21). The effect corresponds to a change in color-index per unit abs. mag. = $0^m.053$, giving a difference of about $0^m.26$ in color-index between a giant and a dwarf of solar type. If $\varepsilon$ is the change of color-index, $\lambda_1$ and $\lambda_2$ the photographic and visual effective wave-lengths, we have for the change in effective temperature.

$$\Delta\left(\frac{1}{T}\right) = \frac{0.4}{c \log e} \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} \varepsilon .$$

If we put $\lambda_1 = 0.43\ \mu$, $\lambda_2 = 0.56\ \mu$, $c = 14300$, $\varepsilon = 0.26$, the effective temperature will change from $T = 6720^0$ to $T' = 5560^0$. The latter value would thus be the effective temperature of a giant of type G 0. Putting $m_0' = 26 m_0$, we have accordingly

$$\frac{5560}{6720} = \frac{t_0'}{t_0}\left(\frac{1 + 2 m_0}{1 + 52 m_0}\right)^{\frac{1}{4}}$$

By putting $\frac{t_0'}{t_0} = 1$ we get

$$m_0 = 0\,024.$$

This value ought to be compared with the values of $\sigma H$ on p. 29; it is nearly identical with the value of $\sigma H$ for $\lambda$ 0.4 $\mu$. Thus for the solar atmosphere we should have $\overline{\varkappa} + \overline{\sigma}$ of the same order of magnitude as $\sigma$ for $\lambda$ 0.4 $\mu$. But of course our values are far too uncertain to serve as a basis for any far-reaching conclusions about the coefficients of absorption and scattering. The value of $m_0$ above ought perhaps to be considered as a minimum value, the values of $\sigma H$ rather as maximum values. The existence of a sensible coefficient of absorption even for relatively thin gases of the solar atmosphere is thus made probable.

2. If we consider all the effective layers, it was made probable from our discussion of the solar radiation that the average coefficient of absorption $\varkappa$ is considerably greater than the coefficient of scattering $\sigma$ through tho whole observable spectrum. From the results just obtained it seems, however, as if this state of things were not necessarily true for the thin atmospheric layers when we enter the violet region of the spectrum.

Thus for the wave-lengths of this region the effect of a strong increase of the atmospheric mass cannot perhaps be given by the general decrease of effective temperature, because our theory about the consequences of the general radiative equilibrium for a separate wave-length rested upon the assumption that $\sigma$ was not great compared with $\varkappa$ for the particular wave-length in question.

If $\varkappa$ and $E$ are now considered imperceptible in the extreme violet for atmospheric layers, we can compute the effect of the general scattering in the giant atmosphere, starting from the formula p. 21, the values of $\sigma H$ on p. 22, and the value $\frac{m_0'}{m_0} = 26$. We find that the intensity at $\lambda$ 0.37 relative to the intensity at $\lambda$ 0.45 $\mu$ would change in the proportion 1:0.75 from a dwarf to a giant of solar type. A change of the effective temperature from 6720° to 5560° would give for the same ratio 1 : 0.81. In this way the change of intensity distribution due to the opalescence scattering may be of a more strongly selective kind than a change of effective temperature.

3. Finally we have to consider the changes of the absorption lines at the transition from a dwarf to a giant, a matter which we have already discussed above. The intensities of most lines of longer wave-lengths ought not to be much changed. But low temperature lines demanding high values of the gas density ought to be weakened and enhanced low pressure lines ought to be strengthened in the giant spectrum — though the giant atmosphere has probably on the whole a somewhat lower temperature than the atmosphere of the dwarf.

An element which occupies a somewhat peculiar position is hydrogen. We may almost consider this element as constituting the upper part of the solar atmosphere, and therefore it is probable that the mass of this element is relatively the most strongly increased one in the giant atmosphere. How far our theoretical arguments are applicable to this element is questionable for several reasons; the observations show that its absorption lines are decidedly strengthened[1] at the transition from dwarf to giant.

But for the effect upon the general distribution of energy in the spectrum we have above all to consider the strong ultra-violet absorption lines, for which we have seen that the extinction probably approaches a pure scattering of the radiation from inner layers. We have seen in the foregoing that the accumulation of these lines is probably sufficiently strong to cause the greatest part of the depression of the solar energy-curve in the ultra-violet region. We have also seen that we may expect here a strong increase of the ultra-violet depression with increasing luminosity.

The ultra-violet portion in the spectra of some bright stars has been investigated especially by W. Huggins[2] and O. Kohl[3]. But a direct verification of our theoretical results by spectrographic investigations using strong dispersion is made difficult by the apparent faintness of the dwarf stars. Further we cannot expect a single ultra-violet line to appear much darker, relative to the surrounding part of the continuous spectrum, in a spectrogram of a giant than in one of a dwarf, for because of the extended wings of the strong lines the continuous radiation ought to be considerably weakened also.

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[1] See W. S. Adams and A. H. Joy, loc. cit.

[2] W. Huggins and Lady Huggins, An Atlas of Representative Stellar Spectra. London 1899.

[3] O. Kohl, Astron. Mitteilungen der Sternwarte zu Göttingen, XVI, 1913.

In the classical comparison between the sun and $\alpha$ Aurigæ, where the relative intensities of the lines seem to agree fairly well far into the ultra-violet, we must not overlook the fact mentioned above that the latter star is a spectroscopic binary. The principal star is of a solar type, whereas, according to W. W. Campbell[1], the secondary star is of a type intermediate between the solar and Sirian types, the former component being only half a magnitude brighter photographically than the bluer component. At least before the crowding of the lines in the Balmer series beyond $\lambda$ 0.370 $\mu$ we must therefore expect the extreme violet part of the giant spectrum of the primary to be perceptibly brightened, and the contrast of the absorption lines against the continuous background diminished, by the presence of the early type component. In fact the absorption lines of both components must fade owing to the superposition of their spectra.

On the other hand, if we compare Huggins' photographs of the solar spectrum with his spectrograms of Arcturus, we find, as Huggins himself points out, a very marked strengthening of the very strong ultra-violet groups of iron lines about $\lambda\lambda$ 388 $\mu\mu$, 382 $\mu\mu$, 374 $\mu\mu$. It seems as if it were mainly this circumstance that causes the weakness in the ultra-violet of Arcturus relative to the sun. (See especially »historical spectra», plate II). Of course this may be thought to depend on the fact that Arcturus is a star of a somewhat »later» stage than the sun, but there are many facts which tend to show that we are here mainly concerned with an effect of absolute magnitude. Some of the lines in the groups are really low temperature lines[2], but by no means all.

In order to examine the matter we need ultra-violet spectrograms of giants and dwarfs of the same spectral type. At least a strong indication that our suspicion mentioned is correct is found in the results about the minimum wave-length mentioned in the introduction. The minimum wave-lengths on Imperial plates have a very marked increase for giants of type G 5 — G 8 and then become almost constant; for dwarfs a decided but much smaller increase of $\lambda_{\min}$ takes place at type K 0. The cause that is at work in the giant series must be very strongly selective, and from these facts alone, if compared with HUGGINS' result mentioned above, it becomes very probable that the ultra-violet iron bands increase up to a relatively much higher intensity in giant spectra of type G — K than in the corresponding dwarf spectra.

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[1] W. W. Campbell, Lick Obs. Bull. No. 6, 1901.

[2] The flame spectrum of iron has been investigated by Ch. de Watteville, Phil. Trans. of the R. Soc. of London, A 204, p. 139, 1904. The most prominent low temperature lines have been tabulated by H. G. Gale and W. S. Adams, Astrophys. Journ. 35, p. 10, 1912; Mt. Wilson Contr. No. 58.

Professor BERGSTRAND has kindly placed at my disposal his effective wave-length plates taken with the great reflector at Meudon in 1908[1]; some important features may be discerned here especially in the grating spectra of the third order. Thus I have compared the two dwarfs of 61 Cygni, types K 7 and K 8, with other stars of nearly the same type which are certainly giants. As a test for equal intensity of image I have taken the appearance of a trace of the spectra of the fifth order. The former stars show a rather regular decrease of blackening towards the air limit at about 300 $\mu\mu$, the latter stars show a large gap beyond the region of H and K, which must probably be ascribed to the influence of the three iron groups. The spectrum ends in a relatively strong point near $\lambda$ 370 $\mu\mu$.

A third very strong indication pointing in the same direction may be obtained from KAPTEYN's discussion[2] of some points in MISS MAURYS' classification. In the series of late spectral types (after the solar type XIV giant types of spectra) there is between types XIV and XV a sudden increase of the absorption beyond $\lambda$ 3889 Å in such a way that for all later types the radiation of shorter wave-lengths becomes difficult to photograph. The degree of this absorption may vary from one star to another, MISS MAURY divides the stars of type XV into two groups, those resembling $\alpha$ Cass. with very strong absorption and those resembling $\alpha$ Bootis with somewhat less conspicuous absorption. There are also in type XIV some stars resembling $\alpha$ Cass. KAPTEYN has collected proper motions of stars belonging to the two groups and finds that the proper motions are on an average much smaller for the first group than for the second, the stars of the $\alpha$ Cass. group have thus a greater mean luminosity than those of the $\alpha$ Bootis group. Some of the stars used by KAPTEYN may be found in the parallax-catalogue of ADAMS and JOY. Twelve stars of type XV,

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[1] See note on p. 6.

[2] J. C. KAPTEYN, Astrophys. Journ. 29, p. 46, 1909; Mt. Wilson Contr. No. 31.

$\alpha$ Cass. group, give the mean values spectral type G 9, abs. mag. + 0.6; thirteen stars of the $\alpha$ Bootis group give G 8, + 1.1. In the latter group a single dwarf of type K 0, abs. mag. + 5.7, has been left out in deriving the mean values. The $\alpha$ Cass. group for solar stars is represented by three giants, mean type G 3, mean abs. mag. + 0.9.

Kapteyn considers his results as an indication of the existence of an absorption of light in space, but from the close coincidence of the wave-length 3889 Å with the limit of the first iron group we are very probably able to state *that the strong ultra-violet iron lines and their wings are very much strengthened at the transition from a dwarf to a giant in the types G and K.*

* * *

We have found above that when we pass from a dwarf to a giant the effect of the increased atmospheric mass above unit surface has produced different results in different spectral regions. For wave-lengths larger than 0.45 $\mu$ we have to reckon with a real change of the effective temperatures of the stars; thus a color-equivalent *C.E.*, determined in this spectral region, will be simply a function of this effective temperature $T$; we shall have

$$C.E. = f(T)\,.$$

When we consider stars of different spectral types aud luminosities, the effective temperature is to be considered as a function of the spectral type $S$ and the absolute magnitude $M$, thus

$$T = \varphi(S, M)\,.$$

For the dwarf series the ultra-violet intensity $I_{uv}$, which we suppose to be measured at the limit of the usual photographic spectrum towards shorter wave-lengths, will be a function of the spectral type $S$ on account of the change of effective temperature, but in addition also by reason of the varying conditions of selective absorption, due to the change of predominant absorbing elements and the varying size of the atmospheres from one type to another. If we consider stars of different luminosities within the types, the ultra-violet intensity will also be a special function of the difference $M - M_0$, where $M_0$ is the absolute magnitude in the dwarf series; the function in question depends on the varying importance of selective scattering. We may put

$$I_{uv} = \psi(T, S, M - M_0) .$$

Thus we shall have

$$\begin{cases} C.E. = A(S, M) \\ I_{uv} \quad = B(S, M) , \end{cases} \quad \ldots \ldots \quad (24)$$

where $A$ and $B$ denote different functions. By combining a color-equivalent determined from relatively long wave-lengths with a measurement of the relative ultra-violet intensity we ought to be able to determine both the spectral class $S$ and the absolute magnitude $M$.

$M$ being known, we get the parallax $\pi$ by means of the formula

$$5 \log \pi = M - m - 5 ,$$

where $m$ is the apparent magnitude.

# III. The two color-equivalents of the grating spectra.

## 1. Examination of stars with known spectral types and luminosities.

In the following I have tried to verify the final conclusions of the preceding chapter by a new examination of the photographic grating spectra for stars of different spectral types and luminosities. The spectra used here have the same general character as those of foregoing investigations with a distance amounting to about 1 mm. between the two spectra of first order. The method followed is essentially the same as in the preliminary notice[1] referred to in the introduction. The effective wave-length $\lambda_c$ is measured by bisecting with the micrometer thread the surfaces of the photographic images of the first order spectra; it is thus defined by the distance between the centra of density in the two spectra. The minimum wave-length $\lambda_{\min}$, defined by the minimum distance between the spectra, is our ultra-violet color-equivalent. Both color-equivalents must be defined for a certain intensity of image, which is measured here by the diameter $D$ of the central image.

We assume for the moment that the spectra are pure, i. e. that each wave-length corresponds to a narrow strip of the photographic spectrum. The law of blackening on the photographic plate we may write in Schwarzschild's[2] form

$$S = \varphi(I \cdot t^p),$$

where $S$ is the blackening, $I$ the intensity of the radiation, $t$ the time of exposure. $p$ is not far from unity and has been found to be essentially independent of $t$. Several investigations have given the result that $p$ is independent of the wave-length[1]; E. Kron[2] has found $p$ variable with the intensity $I$; he puts the law of blackening in the form

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[1] Astrophys. Journ. 49, p. 289, 1919.

[2] K. Schwarzschild, Publ. der v. Kuffnerschen Sternwarte in Wien, Bd. V, C, 1900.

$$S = \varphi(s),\ s = t \cdot \psi(I).$$

We start here from the assumption that for equal blackening on the plate and equal time of exposure $p$ may be considered perfectly constant for all wave-lengths of the photographic region here employed, about $\lambda\lambda\ 0.370 - 0.460\ \mu$. A certain attention ought perhaps to be paid to this question in order to avoid systematic errors; we may get a control by comparing stars with known types but varying apparent magnitudes.

The sensibility-curve of the plate may be defined by the reciprocal values of the different intensities of radiation which are necessary for different wave-lengths in order to give a certain blackening below the maximum density. Because the function $\varphi$ defining the gradation of the plate may vary from one wave-length to another, the sensibility-curve thus defined may perhaps be somewhat different for different degrees of blackening. In order that the relative sensibility for different wave-lengths shall be the same for all plates we must of course employ plates of the same sort and the same emulsion.

Now we must take into consideration the fact that each wave-length does not correspond to a single point on the plate, but that because of the imperfectness of the lenses and the grating each wave-length gives a small surface widening with an increase in the time of exposure; J. Scheiner[3] has shown that the widening of usual photographic images is mainly due to irregularities in the objective and not mainly to a process in the photographic film. By the photographic process the resulting intensity-curves of the three coordinated images finally produce the blackening on the plate. We assume that the boundaries of the central image and the two spectra are given for a certain degree of blackening, then it is evident that *the diameter of the central image determines the contour of the spectral images independently of the general gradation law and the general degree of sensibility of the plate.* The minimum wave-length is thus given essentially independently of these two qualities, which must be considered variable from one plate to another. If we use a sort of plate that gives strong contrasts this will to a large extent be the case also for the effective wave-length, because the maximum density will then be attained rapidly, and differences in the effective wave-length will mainly be due to differences in the form of the images and not to differences of density between different parts of the spectra. In the case of weak images, however, the gradation must play a more important rôle for the effective wave-length; of course the gradiation will also be of some importance for the measurement of the minimum wave-length. In order that the gradation shall not differ too much from one plate to another we ought therefore to employ the same process of development for each plate.

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[1] See bibliography of A. Odencrants, Zeitschrift für wiss. Photographie, Bd. 16, Heft 4, 1916. E. Kron, Publ. des Astrophys. Obs. zu Potsdam, Bd. 22, Nr. 67, 1913, p. 6. H. Rosenberg, loc. cit. in the introduction, p. 77.

[2] E. Kron. loc. cit., p. 54.

[3] J. Scheiner, Photographie der Gestirne, p. 210.

Finally we must consider the influence of occasional disturbances due to the unsteadiness of the optical images during the exposure and to the imperfectness of the plate. The rejection of bad photographic images will be more easy if we get a considerable number of stars in the same exposure and if several exposures with the same time are taken on each plate.

The instrument employed here is the twin 6-inch astrograph of the Upsala Observatory with Zeiss triplet lenses (aperture 15 cm, focal length 150 cm)[1]. The minimum focus lies near $\lambda$ 416 $\mu\mu$. The grating, placed in front of objective I, is furnished by J. L. Rose's mechanical factory at Upsala; it consists of a quadrangular frame of steel over which are stretched parallel steel wires from one side to the opposite one. These wires are drawn through holes bored in a ledge on each of the two sides in such a way that the space between two threads is nearly equal to the breadth of a thread. A preliminary measurement gave the grating constant $c$ to be on an average 1.2605 mm.; by means of this value the wave-lengths in the following tables are calculated. A more rigorous determination gave the mean value $c = 1.2595$ mm., and the breadth of a thread on an average 0.62 mm.; because of the irregularities of the grating a small correction to the new value of $c$ would be quite useless. If $s$ is the distance measured on the plate, $f$ the focal distance, and if we suppose the inclination of the rays towards the normals of the grating and of the plate to be zero, we have the corresponding wave-length $\lambda$ by the relation

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[1] The same instrument as in the previous investigations of this kind made at Upsala which are mentioned in the introduction.

$$\lambda = \frac{c}{2f} \cdot s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

For $f$ I have used the value 1485.5 mm. More recent measurements gave 1484.4 mm.; the difference between these values is of no importance here, because we may disregard very small errors which are constant for all stars.

In this investigation I have not used exactly the center of the plate but a region giving somewhat more regular images. For this reason the ocular of the guiding tube was moved 25 mm. in a direction making an angle of 45° with the hour circle, towards increasing R. A. and Decl. as reckoned on the image of the plate (tube following). The deviations from equation (25) caused by this arrangement will be very small and of no importance, because neither the plate nor the grating is perfectly centrated. Only plate 1 in table 8 is taken with the ocular moved 20 mm. towards increasing declination of the plate image. The grating was placed with the threads in declination, the grating spectra are consequently situated in R. A. This arrangement was found to give more symmetrical images than the one with the spectra in declination.

The plate is situated at most some tenths of a millimeter outside the minimum focus; the images of the spectra are very sharp and show a smooth distribution of blackening even for faint images. With increasing distance from the objective two condensations of density are shown in the spectra corresponding to the two wave-lengths in the foci of which the plate is situated. The appearance of such condensations would change perceptibly the conditions for the minimum wave-length and must be avoided here. Most plates used here were taken in the spring and autumn with a rather small variation of temperature, but some of the first and latest plates in table 8 were taken at considerably lower temperatures. No decided change in the appearance of the images of these plates was discovered.

The sort of plates used are Kranz I, Orthochromatische Momentplatte, Kranseder, München. An investigation of Mr Ossian Vallin's has shown that this sort of plates gives extremely sharp and well defined images compared with a great number of other sorts of plates. This plate is moreover found to be very sensitive, though accumulations of the silver-grains are very slightly conspicuous when the plate is looked upon through the microscope of the measuring-machine. Thus far this plate is therefore very suitable for a study of the grating spectra.

An investigation of the sensibility-curves for some different sorts of plates has been carried out according to a method of A. Odencrants. For this investigation Professor G. Granqvist has kindly permitted me to use the photographic labarotory of the Physical Institution at Upsala, and I am especially indebted to Dr. Odencrants for his assistance and valuable advice. It was found that the sensibility of the Kranz plate is relatively much smaller between the G and F line than that of an Imperial plate; the former plate has a small secondary maximum at about $\lambda$ 550 $\mu\mu$, but this is of no importance in our case; no trace of this maximum can be discerned in images of grating spectra which are not very much over-exposed. The region of sensibility thus being smaller we must expect a priori that the variation of $\lambda_c$ between different spectral classes should be perceptibly smaller for Kranz than for Imperial, and this has been established, as will be shown in the following. However, the very great accuracy with which $\lambda_c$ may be measured on the Kranz plate probably outweighs this less favourable circumstance. The diameter of the central image, which serves to define the intensity of images, may be measured here with very great accuracy, which is necessary for the determination of $\lambda_{\min}$.

The stars examined here are generally taken from the parallax-catalogue of Adams and Joy; a number of exposures with varying times were secured of each star. Generally stars of different types and luminosities were taken on the same plate. With objective II I have generally obtained simultaneously images on an orthochromatic plate (Hauff's Flavin) behind yellow-filter in order to register the intensity in a third region of the spectrum, but these plates have not been used here; their reduction is made difficult by the fact that we do not know if SCHWARZSCHILD's exponent is the same for both sorts of plates.

As a developer I have used Rodinal, 6 cm$^3$ in 120 cm$^3$ water with ten drops of $KBr$, temperature of the bath 18$^0$, time of development 8 min. The time of development was one to two minutes shorter for the three first plates in table 8.

The measurements have been performed by means of the Repsold measuring machine of the observatory. The effective wave-length was measured in the same way as has been described in previous investigations. The minimum wave-length was measured here with a single thread of the micrometer in the same way as the central diameter $D$. After a plate was measured in one position it was turned 180$^0$ and measured again. In each position several measurements of the three coordinated quantities were carried out. The measurement of $\lambda_{\min}$ and $D$ must be performed with great care, the thread is moved slowly till it touches slightly the border of the images. It is possible that one's idea of the boundaries of the images, perhaps also the manner of moving the thread, may be subject to slight changes, especially at the beginning of the measurements. We may, for instance, to begin with, demand a higher degree of blackening at the border, we then measure $D$ smaller and $\lambda_{\min}$ larger than in the following measurements. If the deviations are small these changes will probably nearly compensate each others, the effect will be the same as if we measure at a later time an image with a somewhat shorter exposure. If the deviations are larger the errors introduced will probably not correspond exactly because of the different character of the images, nor will they do so if they are due to a different manner of measuring. The first measurements here were performed in the spring of 1919, the exposures and measurements were then continued in the autumn and winter of 1919—1920. By repeating the measurements for some stars I have found a perceptible difference in the measurements between these two periods and have therefore reduced the first measurements (plates 1—7) by adding 1.0 to the measured value of $D$ and subtracting 3.0 $\mu\mu$ from $\lambda_{\min}$. Reckoned in revolutions of the micrometer the two corrections are nearly the same, $0^R.01 = 0.005$ mm. In the final values of $\lambda_{\min}$ this correction is nearly equivalent to subtracting 1.5 $\mu\mu$. This correction concerns stars of different types and luminosities — in all about one third of the known stars examined. In the case of the effective wave-length no perceptible differences were given by repeated measurements.

The measurements are made difficult if there is a perceptible veil of blackening on the plate, several plates taken have been excluded for this reason. For very short exposures, smaller than 10 sec., a rigorous control must be applied to the quality of the images because the deviations caused by trembling of the instrument and by atmospheric refraction will have a more irregular influence here than for a comparatively long exposure. Exposures exceeding 1 min. generally give images which fulfil all demands for symmetry and sharpness.

Table 8 contains a list of the plates used in this investigation. The third column contains the average temperature of the air, registered at the Meteorological Observatory, which is situated in the immediate neighbourhood. The temperature of the instrument is to be estimated some degrees higher. The fourth column contains the stars on the plate, the numbers refer to the first column of table 9. All stars are taken in a position less than $3^h\ 30^m$ from the meridian, generally less than $1^h$.

Table 8.

| Plate number | Date | Temperature | Stars | Remarks |
|---|---|---|---|---|
| 1 | Feb. 26, 1919 | − 12° C | 1—10 | |
| 2 | March 5 | − 5 | 11—18 | Some haze especially for star 18. |
| 3 | March 6 | − 2 | 19 | |
| 4 | March 8 | − 5 | 20, 21 | |
| 5 | March 15 | − 4 | 19 | Some haze. |
| 6 | April 22 | + 3 | 22 | |
| 7 | April 26 | − 1 | 23—27 | |
| 8 | August 26 | + 7 | 28—34 | Some hase at the and. |
| 9 | Sept. 1 | + 9 | 35—40 | |
| 10 | Sept. 25 | + 7 | 41—43 | |
| 11 | Sept. 25 | + 4 | 44 | |
| 12 | Oct. 4 | + 8 | 45—47 | Air unsteady. |
| 13 | Oct. 4 | + 7 | 48-50, 62 | Air unsteady, especially for star 62, which has been excluded here. |
| 14 | Oct. 31 | − 4 | 51, 52 | |
| 15 | Nov. 10 | − 13 | 53—56 | Air very steady. Some veil on the plate. |
| 16 | Nov. 14 | − 12 | 57, 58 | Air unsteady, trace of fog. |
| 17 | Dec. 11 | − 8 | 57—61 | |
| 18 | Jan. 9, 1920 | − 4 | 62, 63 | Air not quite steady. |

The results of the measurements are given in table 9, where the stars from the same plate are tabulated together in the order in which they have been taken. The two last columns contain $\lambda_c$ and $\lambda_{\min}$ reduced to the normal diameter $D = 17$ by means of the curves in figures 4 and 5. These values are given as means of the measurements on images with the same or nearly the same time of exposure, $\lambda_c$ and $\lambda_{\min}$ are expressed in $\mu\mu$, $D$ in hundredths of a revolution of the micrometer. $1^R$ corresponds to 0.505 mm. The time of exposure is given in minutes or seconds.

Table 9.

| Star | | Exposure | $\lambda_c$ | $\lambda_{\min}$ | $D$ | $D = 17$ | |
|---|---|---|---|---|---|---|---|
| | | | | | | $\lambda_c$ | $\lambda_{\min}$ |
| | | | $\mu\mu$ | $\mu\mu$ | | $\mu\mu$ | $\mu\mu$ |
| 1 | Lal. 18115[1] | $10^m$ | 423.9 | 375.6 | 16.7 | 423.6 | 376.4 |
| | | 10 | 22.6 | 79.0 | 16.1 | | |
| | | 20 | 27.4 | — | 20.3 | 24.8 | — |
| 2 | Lal. 18115[2] | $5^m$ | 423.3 | — | 14.2 | 424.5 | — |
| | | 10 | 24.5 | — | 16.5 | 25.6 | — |
| | | 10 | 26.7 | 378.3 | 17.2 | | 378.7 |
| | | 20 | 26.0 | 73.5 | 18 8 | 24.7 | 76.5 |
| 3 | 83 Cancri | $1^m$ | 414.4 | — | 15.6 | 414.8 | — |
| | | 3 | 20.5 | 365.5 | 21.3 | 16.8 | 374.3 |
| | | 3 | 18.7 | 65.3 | 20.9 | | |
| | | 5 | 21.3 | 61.0 | 23.7 | 16.0 | — |
| 4 | $\mu$ Urs. maj. | $5^s$ | 428.4 | 387.2 | 14.3 | 429.0 | 383.4 |
| | | 5 | 29.5 | — | 15.2 | | — |
| | | 10 | 31.8 | 83.9 | 17.0 | 30.5 | 82.3 |
| | | 10 | 29.2 | 80.1 | 17.3 | | |
| | | 20 | 29.1 | 76.2 | 21.1 | 29.2 | 82.5 |
| | | 31 | 31.8 | 76.4 | 21.6 | | |
| | | 40 | 35.7 | 67.6 | 24.9 | 30.3 | — |
| | | 40 | 33.4 | 70.6 | 25.2 | | — |
| | | 60 | 36.6 | 65.3 | 27.6 | — | — |
| 5 | Boss 2921 | $2^m$ | 433.1 | 384.6 | 14.9 | 433.1 | 381.6 |
| | | 5 | 30.1 | 75.8 | 21.3 | 28.9 | 82.0 |
| | | 10 | 33.1 | 66.5 | 24.3 | 28.4 | — |
| 6 | Lal. 21185 | $5^m$ | 423.8 | — | 13.7 | 425.0 | — |
| | | 10 | 23.1 | 377.7 | 17.8 | 22.6 | 379.0 |
| | | 16 | 24.6: | 73.0 | 19.5 | 22.7 | 77.3 |
| 7 | Boss 3089 | $20^s$ | 429.5 | 386.3: | 14.9: | 428.7 | 380.8 |
| | | 20 | 27.8 | 79.4: | 16.1 | | |
| | | 30 | 27.8 | — | 17.5 | 27.8 | — |

| Star | | Exposure | $\lambda_c$ | $\lambda_{min}$ | $D$ | $D = 17$ | |
|---|---|---|---|---|---|---|---|
| | | | | | | $\lambda_c$ | $\lambda_{min}$ |
| | | | $\mu\mu$ | $\mu\mu$ | | $\mu\mu$ | $\mu\mu$ |
| | | $1^{m}$ | 429.7 | 376.6 | 21.7 | 428.3 | 384.4 |
| | | 1 | 30.3 | 78.8 | 21.5 | | |
| | | 1 | 29.1 | 77.9 | 21.7 | | |
| | | $2^{m}$ | 32.9 | 73.8 | 26.2 | — | — |
| | | 3 | 38.9 | 70.0 | 27.9 | — | — |
| 8 | δ Virg. | $20^{s}$ | 427.5 | 379.6 | 17.2 | 427.5 | 379.9 |
| | | 30 | 27.5 | 73.6 | 20.7 | 27.1 | 80.6 |
| | | 40 | 30.3 | 74.9 | 21.1 | | |
| | | 40 | 27.3 | 74.3 | 22.4 | | |
| | | 90 | 38.3 | 67.0 | 25.3 | — | — |
| 9 | ε Virg. | $3^{s}$ | 418.1 | 381.2 | 14.1 | 420.0 | 377.2 |
| | | 3 | 19.0 | — | 15.0 | | — |
| | | 5 | 21.3 | — | 17.8 | 20.7 | — |
| | | 10 | 22.4 | — | 19.7 | 20.2 | — |
| | | 20 | 27.6 | 65.0 | 24.3 | — | — |
| | | 40 | 34.0 | 57.5 | 29.0 | — | — |
| 10 | Boss 3398 | $2^{m}$ | 423.7 | 380.9 | 16.6 | 423.8 | 380.4 |
| | | 5 | 28.4 | 69.8 | 22.0 | 24.3 | 77.0 |
| | | 8 | 26.9 | 65.7 | 24.3 | — | — |
| 11 | o Leon. | $3^{s}$ | 412.6 | 382.4 | 12.5 | 417.3 | — |
| | | 3 | 16.0 | — | 12.1 | | — |
| | | 12 | 15.8 | 73.8 | 16.1 | 18.2 | 373.8 |
| | | 12 | 19.0 | 77.2 | 16.0 | | |
| | | 20 | 17.5 | 68.2 | 18.1 | 20.0 | 72.1 |
| | | 30 | 23.7 | 69.3 | 19.1 | | |
| 12 | Boss 2603 | $5^{m}$ | 423.3 | 381.9 | 14.3 | 424.3 | 378.4 |
| | | 5 | 23.9 | 82.1 | 14.5 | | |
| | | 10 | 24.8 | 77.4 | 18.1 | 24.2 | 78.9 |
| 13 | Lal 19821 | $2^{m}.5$ | 417.5 | — | 10.3 | — | — |
| | | 5 | 15.3 | 379.8 | 12.4 | 418.2 | — |
| | | 10 | 17.3 | 78.5 | 14.0 | 18.8 | 373.6 |
| 14 | 39 Leon. | $30^{s}$ | 417.5 | — | 14.3 | 418.7 | — |
| | | 30 | 16.8 | 379.4 | 13.8 | | 374.3 |
| | | $1^{m}$ | 15.5 | 73.7 | 16.7 | 16.0 | 75.4 |
| | | 1 | 16.4 | 78.5 | 17.1 | | |
| | | 3 | 18.1 | 67.6 | 20.6 | 15.8 | 75.4 |
| 15 | χ Leon. | $8^{s}$ | 414.7 | — | 13.2 | 416.8 | — |
| | | 8 | 14.9 | — | 13.5 | | — |
| | | 15 | 17.0 | 377.4 | 15.1 | 17.1 | 374.0 |
| | | 15 | 15.8 | 77.0 | 15.2 | | |
| | | 30 | 18.8 | 69.9 | 18.3 | 16.9 | 72.4 |
| | | 30 | 15.8 | 69.9 | 18.0 | | |
| | | $1^{m}$ | 20.3 | 64.0 | 20.9 | 17.7 | 73.2 |
| | | 1 | 19.8 | 66.3 | 20.5 | | |

| Star | | Exposure | $\lambda_c$ | $\lambda_{\min}$ | $D$ | $D = 17$ | |
|---|---|---|---|---|---|---|---|
| | | | | | | $\lambda_c$ | $\lambda_{\min}$ |
| | | | $\mu\mu$ | $\mu\mu$ | | $\mu\mu$ | $\mu\mu$ |
| 16 | 46 Leo min | $6^s$ | 422.0 | 379.2 | 13.1 | 421.6 | 375.5 |
| | | 6 | 17.9 | 82.6 | 13.3 | | |
| | | 10 | 21.3 | 77.2 | 15.1 | 21.8 | 74.7 |
| | | 10 | 20.3 | 76.2 | 15.6 | | |
| | | 20 | 20.3 | 75.1 | 17 6 | 19.0 | 76.7 |
| | | 20 | 18.8 | 73.6 | 18.1 | | |
| | | 40 | 24.5 | 70.4 | 20.7 | — | — |
| | | 60 | 25.2 | 59.0 | 24.2 | — | — |
| 17 | Groom. 1646 | $1^m$ | 416.2 | 377.5 | 11.0 | 420.3 | — |
| | | 2 | 19.2 | 75.1 | 14.7 | 19.8 | 373.5 |
| | | 3 | 19.0 | 77.9 | 15.1 | | |
| | | 3 | 19.2 | 77.0 | 15.9 | | |
| | | 8 | 21.1 | 68.2 | 19.9 | 19.5 | 74.4 |
| 18 | Groom. 1596 | $4^m27^s$ | 419.0 | 377.2 | 13.7 | 420.8 | 372.0 |
| | | $20^m$ | 20.9 | 68.7 | 19.2 | 19.8 | 73.3 |
| 19 | $\theta^1$ Tauri | $3^s$ | 420.3 | 382.8 | 13.4 | 421.9 | 377.7 |
| | | 10 | 19 4 | 75.7 | 16.9 | 20.8 | 75.6 |
| | | 10 | 22.2 | — | 17.1 | | — |
| | | 30 | 24.5 | — | 22.4 | — | — |
| 20 | $c$ Plej. | 3 | 410.6 | 368.4 | 17.0 | 410.6 | 368.4 |
| | | 10 | 12.7 | 62.2 | 21.5 | 11.1 | 70.9 |
| | | 10 | 14.0 | — | 21.8 | | |
| | | 20 | 14.4 | 55.8 | 23.9 | 11.4 | 70.0 |
| | | 20 | 15.5 | 56.7 | 24.0 | | |
| | | 30 | 15.7 | 52.0 | 26.3 | — | — |
| 21 | $e$ Plej. | $3^s$ | 410.4 | 366.7: | 14.0 | 411.0 | — |
| | | 3 | 09.5 | 68.9 | 15.7 | | 366.4 |
| | | 3 | 11.0 | 68.0 | 16.6 | | |
| | | 10 | 10.6 | 64.4 | 20.1 | 09.2 | 70.8 |
| | | 20 | 15.1 | 57.3 | 23.4 | 11.8 | 69.5 |
| | | 30 | 15.7 | 54.3 | 25.2 | 11.3 | — |
| | | $1^m$ | 17.4 | 45.2 | 28.0 | — | — |
| | | 2 | 19.1 | 39.3 | 31.5 | — | — |
| 19 | $\theta^1$ Tauri | $3^s$ | 418.8 | 381.7 | 11.7 | 420.7 | 375.7 |
| | | 3 | 19.8 | 85.6 | 11.7 | | |
| | | 10 | 19.0 | 81.7 | 14.6 | 20.5 | 76.9 |
| | | 10 | 20.0 | 78.1 | 16.1 | | |
| | | 20 | 21.8 | 74.0 | 18.7 | 20.6 | 78.1 |
| 22 | $\varrho$ Bootis | $10^s$ | 423.5 | 390.0: | 12.0 | 424.8 | — |
| | | 40 | 26.0 | 78.7 | 19.0 | 24.8 | 381.3 |
| | | 60 | 26.0 | 80.6 | 20.5 | 23.6 | 85.5 |
| 23 | $\eta$ Leon. | $1^s$ | 411.7 | 374.9: | 13.4 | 414.8 | 371.4 |
| | | 1 | 09.7 | 75.1 | 14.3 | | |
| | | 1 | 10.6 | — | 12.6 | | |

| Star | | Exposure | $\lambda_c$ | $\lambda_{\min}$ | $D$ | $D = 17$ | |
|---|---|---|---|---|---|---|---|
| | | | | | | $\lambda_c$ | $\lambda_{\min}$ |
| | | | $\mu\mu$ | $\mu\mu$ | | $\mu\mu$ | $\mu\mu$ |
| | | 2 | 409.3 | — | 14.9 | 411.6 | — |
| | | 3 | 12.5 | 475.9 | 16.2 | | 71.9 |
| | | 4 | 14.0 | 68.9 | 16.9 | 14.0 | |
| | | 4 | 14.7 | — | 16.9 | | — |
| | | 6 | 14.2 | 69.5 | 19.2 | 12.8 | 72.0 |
| | | 7 | 16.4 | 67.4 | 20.0 | 14.8 | 70.3 |
| | | 10 | 16.8 | 62.7 | 20.3 | | |
| | | 15 | 16.4 | 58.2 | 22.7 | 13.8 | 69.6 |
| | | 20 | 20.2 | — | 23.9 | 13.1 | — |
| | | 20 | 18.3 | 53.0 | 25.2 | | — |
| | | 30 | 20.9 | 50.7 | 27.0 | — | — |
| 24 | Lal. 25224 | $10^s$ | 413.8 | 375.9 | 12.3 | — | — |
| | | 15 | 13.4 | 74.9 | 13.5 | 417.4 | 369.7 |
| | | 25 | 15.1 | 71.4 | 15.0 | 16.9 | 69.1 |
| | | 25 | 14.9 | 70.1 | 15.2 | | |
| | | 30 | 14.9 | 72.3 | 15.5 | | |
| | | 30 | 16.6 | 70.6 | 16.8 | 16.8 | 70.4 |
| | | $1^m$ | 18.1 | 69.5 | 18.3 | 17.1 | 70.6 |
| 25 | β Comæ | $5^s$ | 414.4 | 378.3 | 13.8 | 418.1 | 373.6 |
| | | 5 | 18.3 | 78.5 | 14.3 | | |
| | | 10 | 18.1 | 76.1 | 16.1 | 18.3 | 73.3 |
| | | 10 | 17.6 | 74.6 | 15.6 | | |
| | | 16 | 18.5 | 71.2 | 17.0 | 18.3 | 71.8 |
| | | 16 | 18.3 | 70.4 | 18.0 | | |
| | | 30 | 21.1 | 65.9 | 20.4 | 19.0 | 73.2 |
| | | 45 | 23.2 | 60.5 | 22.9 | 18.7 | 73.5 |
| 26 | Lal. 24652 | $1^m$ | 421.7 | 384.9 | 11.6 | 423.0 | 377.3 |
| | | 2 | 21.8 | 82.8 | 14.1 | 23.0 | 78.6 |
| | | 3 | 22.4 | 79.1 | 16.5 | 22.7 | 78.3 |
| | | 5 | 23.6 | 75.1 | 17.9 | 22.8 | 77.8 |
| | | 5.5 | 23.4 | 77.2 | 18.1 | | |
| | | 8 | 24.6 | 73.8 | 19.9 | 22.3 | 78.8 |
| 27 | Boss 3605 | $3^m$ | 429.2 | 385.1 | 16.1 | 429.8 | 383.7 |
| | | 3 | 30.3 | 83.6 | 16.8 | | |
| 28 | σ Drac. | $10^s$ | 418.5 | 376.5 | 15.6 | 419.0 | 374.1 |
| | | 20 | 18.9 | 72.5 | 17.7 | 18.8 | 74.1 |
| | | 20 | 18.9 | 73.7 | 17.0 | | |
| | | 30 | 21.1 | — | 18.8 | 20.3 | — |
| | | 40 | 23.4 | 64.5 | 21.3 | 21.1 | 74.4 |
| | | 40 | 22.7 | — | 21.3 | | |
| | | $1^m$ | 26.6 | 64.1 | 21.8 | — | — |
| | | 2 | 28.1 | 56.2 | 25.6 | — | — |
| | | 3 | 30.5 | 49.1 | 28.4 | — | — |
| 29 | Lal. 38287 | $2^m$ | 418.0 | 383.1 | 13.9 | 419.6 | 378.1 |

| Star | | Exposure | $\lambda_c$ | $\lambda_{\min}$ | $D$ | $D = 17$ | |
|---|---|---|---|---|---|---|---|
| | | | | | | $\lambda_c$ | $\lambda_{\min}$ |
| | | | μμ | μμ | | μμ | μμ |
| | | 4 | 419.7 | — | 17.1 | 419.7 | — |
| | | 6 | 21.9 | 369.2 | 19.3 | 420.8 | 374.0 |
| | | 6 | 21.0 | 71.6 | 18.1 | | |
| | | 12 | 22.3 | 60.0 | 22.5 | 18.2 | 72.1 |
| 30 | 24 Vulpec. | $20^s$ | 419.5 | 377.6 | 14.8 | 420.8 | 374.6 |
| | | 30 | 21.9 | 76.7 | 15.4 | 22.1 | 76.7 |
| | | 30 | 20.8 | 79.3 | 16.3 | | |
| | | 45 | 21.2 | 75.6 | 17.4 | 20.9 | 76.7 |
| | | $1^m$ | 21.5 | 76.2 | 17.3 | 21.0 | 76.4 |
| | | 1 | 20.0 | 79.2 | 16.1 | | |
| | | 2 | 25.1 | 70.5 | 19.7 | 22.9 | 76.8 |
| | | 4 | 28.1 | 56.4 | 26.9 | — | — |
| 31 | $\gamma$ Cygni | $1^s$ | 417.6 | 379.1 | 16.2 | 418.5 | 378.4 |
| | | 1 | 18.0 | — | 15.5 | | |
| | | 1 | 18.0 | 79.5 | 16.3 | | |
| | | 2 | 21.2 | — | 19.3 | 18.8 | — |
| | | 2 | 19.1 | — | 18.7 | | — |
| | | 4 | 22.3 | — | 21.9 | — | — |
| | | 6 | 21.4 | 61.8 | 23.2 | — | — |
| 32 | $61^1$ Cygni | $20^s$ | 425.6 | 384.7 | 12.2 | 427.0 | 377.9 |
| | | 30 | 24.1 | 80.9 | 12.7 | 25.3 | 75.9 |
| | | 30 | 23.9 | 80.9 | 14.2 | | |
| | | $1^m$ | 24.7 | 78.9 | 16.5 | 24.5 | 77.7 |
| | | 1 | 24.3 | 75.9 | 17.3 | | |
| | | 1.5 | 28.0 | 74.4 | 18.5 | 27.0 | 77.0 |
| | | 2 | 26.0 | 67.2 | 21.2 | 23.6 | 77.4 |
| | | 2 | 26.7 | 69.5 | 21.5 | | |
| | | 3 | 29.3 | 69.7 | 21.1 | | |
| | | 4 | 32.9 | 65.5 | 24.2 | 26.2 | — |
| | | 6 | 35.0 | 60.7 | 25.8 | — | — |
| | | $12^m45^s$ | 38.9 | 51.5 | 31.1 | — | — |
| 33 | $61^2$ Cygni | $30^s$ | 423.9 | — | 10.8 | 424.9 | — |
| | | 30 | 23.9 | — | 10.0 | | — |
| | | $1^m$ | 23.7 | — | 13.3 | 23.8 | — |
| | | 1 | 21.1 | 385.1 | 13.3 | | 379.7 |
| | | 1.5 | 24.8 | 83.4 | 14.1 | 26.0 | 79.2 |
| | | 2 | 26.5 | 72.9 | 15.9 | 25.5 | 74.4 |
| | | 2 | 24.1 | 77.4 | 16.2 | | |
| | | 3 | 25.0 | 75.9 | 16.7 | | |
| | | 4 | 27.1 | 69.7 | 19.4 | 25.8 | 74.9 |
| | | 6 | 28.6 | 70.8 | 21.0 | | |
| | | $12^m45^s$ | 32.2 | — | 24.1 | 25.6 | — |
| 34 | $\zeta$ Cygni | $2^s$ | 420.7 | 385.1 | 10.1 | — | — |
| | | 4 | 20.0 | 84.9 | 11.9 | 421.4 | 377.3 |

| Star | | Exposure | $\lambda_c$ | $\lambda_{\min}$ | $D$ | $D = 17$ | |
|---|---|---|---|---|---|---|---|
| | | | | | | $\lambda_c$ | $\lambda_{\min}$ |
| | | | $\mu\mu$ | $\mu\mu$ | | $\mu\mu$ | $\mu\mu$ |
| | | 6 | 420.0 | 381.9 | 14.4 | 421.2 | 377.1 |
| | | 6 | 19.4 | 80.9 | 13.3 | | |
| | | 8 | 17.5 | — | 14.5 | 21.3 | — |
| | | 8 | 22.4 | — | 15.0 | | — |
| | | 12 | 21.5 | 78.5 | 17.5 | 21.9 | 78.4 |
| | | 12 | 22.2 | 78.3 | 16.4 | | |
| | | 20 | 24.1 | 71.9 | 18.7 | 22.9 | 76.1 |
| | | 40 | 29.3 | 67.6 | 22.1 | — | 78.9 |
| | | 60 | 29.5 | 61.0 | 23.1 | — | — |
| 35 | 109 Herc. | $5^{s}$ | 427.8 | 380.6: | 13.4 | 427.8 | — |
| | | 10 | 29.5 | 81.3 | 16.0 | 27.5 | 380.0 |
| | | 10 | 25.4 | — | 15.8 | | — |
| | | 15 | 28.2 | 77.0 | 17.8 | 26.9 | 78.8 |
| | | 20 | 26.3 | — | 18.6 | | |
| | | 20 | 26.3 | 77.4 | 18.5 | | |
| | | 40 | 30.3 | 71.4 | 21.8 | 28.7 | 78.3 |
| | | $1^{m}$ | 30.3 | 67.4 | 25.1 | — | — |
| 36 | $\beta$ Delph. | $5^{s}$ | 416.8 | 373.2 | 18.5 | 416.2 | 376.3 |
| | | 10 | 19.2 | 65.2 | 21.7 | 15.6 | 73.6 |
| | | 10 | 17.7 | 62.2 | 20.9 | | |
| | | 20 | 23.7 | 57.7 | 24.2 | 17.9 | — |
| 37 | Lal. 38683 | $1^{m}$ | 422.4 | 384.9 | 11.7 | 423.7 | 377.5 |
| | | 2 | 21.1 | 81.7 | 14.4 | 22.1 | 77.9 |
| | | 5 | 22.0 | 75.3 | 17.5 | 21.7 | 76.1 |
| | | 12 | 23.7 | — | 20.6 | 20.8 | — |
| 38 | Groom. 3150 | $10^{s}$ | 414.3 | 378.1: | 11.3 | 418.4 | — |
| | | 10 | 15.1 | 76.6: | 11.7 | | — |
| | | 20 | 17.3 | 79.6 | 14.0 | 18.8 | 374.7 |
| | | 40 | 19.2 | — | 17.5 | | — |
| | | 50 | 17.5 | 72.3 | 17.8 | 17.5 | |
| | | 50 | 16.4 | 73.4 | 17.7 | | 74.3 |
| | | 80 | 17.9 | 68.0 | 19.6 | 18.0 | 73.5 |
| | | 80 | 20.5 | — | 19.2 | | — |
| | | $2^{m}$ | 20.9 | — | 21.8 | 17.5 | — |
| 39 | $\tau$ Cygni | $2^{s}$ | 416.2 | — | 14.3 | 417.5 | — |
| | | 4 | 17.5 | 371.0 | 17.5 | 18.1 | 372.0 |
| | | 4 | 18.8 | — | 16.8 | | — |
| | | 5 | 16.4 | 71.0 | 18.2 | 16.3 | 72.0 |
| | | 7 | 17.3 | 66.3 | 18.9 | | |
| | | 10 | 18.8 | 63.7 | 21.2 | | 72.9 |
| | | 10 | 19.8 | — | 21.6 | 16.2 | — |
| | | 10 | 18.3 | — | 20.4 | | — |
| | | 30 | 25.4 | 53.0 | 26.3 | — | — |
| | | 30 | 24.5 | 54.3 | 25.9 | — | — |

| Star | | Exposure | $\lambda_c$ | $\lambda_{\min}$ | $D$ | $D = 17$ | |
|---|---|---|---|---|---|---|---|
| | | | | | | $\lambda_c$ | $\lambda_{\min}$ |
| | | | $\mu\mu$ | $\mu\mu$ | | $\mu\mu$ | $\mu\mu$ |
| 40 | $\eta$ Peg. | $2^s$ | 418.1 | 381.9 | 14.8 | 419.4 | 378.9 |
| | | 5 | 20.3 | 74.9 | 17.7 | 19.8 | 76.7 |
| | | 8 | 24.3 | — | 20.6 | 21.4 | — |
| | | 8 | 24.5 | — | 20.4 | | — |
| | | 12 | 25.0 | 64.6 | 22.4 | — | 75.3 |
| | | 30 | 31.2 | 52.6 | 28.1 | — | — |
| 41 | A. G. Ber. 8366 | $20^m$ | 419.6 | 377.2 | 15.5 | 420.2 | 374.4 |
| | | 30 | 19.4 | 73.2 | 17.8 | 19.1 | 74.8 |
| 42 | Boss. 5602 | $2^m$ | 426.9 | 379.2 | 18.5 | 426.9 | 381.2 |
| | | 3 | 27.1 | 76.2 | 20.4 | 26.5 | 81.0 |
| | | 5 | 30.7 | 72.5 | 21.5 | 26.7 | 79.0 |
| | | 10 | 33.7 | 64.6 | 27.1 | — | — |
| | | 20 | 38.5 | 58.0 | 29.8 | — | — |
| 43 | $\varkappa$ Peg. | $3^s$ | 411.7 | 374.7 | 15.0 | 413.8 | 372.2 |
| | | 6 | 14.1 | — | 18.5 | 13.8 | — |
| | | 6 | 16.0 | 70.4 | 19.3 | | 73.2 |
| | | 10 | 17.7 | 66.1 | 20.7 | 15.8 | 72.5 |
| | | 15 | 20.9 | 58.6 | 23.3 | 17.9 | 71.3 |
| | | 30 | 23.3 | 52.4 | 26.9 | — | — |
| 44 | $\varrho$ Persei | $8^s$ | 429.1 | 386.0 | 16.3 | 427.2 | 382.8 |
| | | 8 | 26.1 | 83.9 | 16.1 | | |
| | | 8 | 26.3 | 81.5 | 16.3 | | |
| | | 15 | 28.1 | 81.7 | 19.3 | 28.2 | 84.6 |
| | | 15 | 28.5 | 82.6 | 18.2 | | |
| | | 30 | 30.6 | 74.9 | 23.0 | 28.2 | 84.9 |
| | | 30 | 32.1 | 76.8 | 23.4 | | |
| | | $1^m$ | 36.9 | 68.0 | 26.6 | — | — |
| | | 1 | 34.1 | 67.4 | 26.3 | — | — |
| | | 3 | 42.2 | 56.6 | 33.1 | — | — |
| 45 | B. D. $26^0$4721 | $5^m$ | 439.4 | — | 12.9 | 439.4 | — |
| | | 10 | 40.7 | — | 15.2 | 38.5 | — |
| | | 10 | 36.2 | — | 15.1 | | — |
| | | 15 | 37.3 | 385.6 | 16.7 | 37.3 | 385.2 |
| 46 | 85 Peg. | $15^s$ | 418.8 | 377.4 | 14.4 | 418.0 | 371.9 |
| | | 15 | 15.2 | 74.2 | 15.1 | | |
| | | 30 | 19.3 | 70.6 | 17.1 | 18.6 | 70.8 |
| | | 30 | 17.8 | 71.0 | 16.8 | | |
| | | $1^m$ | 18.6 | 68.7 | 19.2 | 18.1 | 73.8 |
| | | 1 | 20.6 | 67.4 | 20.2 | | |
| | | 3 | 21.4 | 56.2 | 24.6 | 15.2 | — |
| | | 5 | 27.0 | 47.6 | 27.6 | — | — |
| 47 | $\varepsilon$ Andr. | $8^s$ | 422.5 | 377.4 | 15.9 | 421.1 | 376.5 |
| | | 8 | 21.8 | — | 16.7 | | |
| | | 8 | 21.0 | 76.6 | 17.3 | | |

| Star |  | Exposure | $\lambda_c$ | $\lambda_{\min}$ | $D$ | $D \doteq 17$ |  |
|---|---|---|---|---|---|---|---|
|  |  |  |  |  |  | $\lambda_c$ | $\lambda_{\min}$ |
|  |  |  | $\mu\mu$ | $\mu\mu$ |  | $\mu\mu$ | $\mu\mu$ |
|  |  | 10 | 419.3 | 377.2 | 17.0 | 420.1 | 374.9 |
|  |  | 15 | 20.6 | 70.6 | 18.8 |  |  |
|  |  | 15 | 21.0 | 71.0 | 18.8 |  |  |
|  |  | 15 | 23.6 | 69.7 | 18.0 |  |  |
|  |  | 15 | 20.1 | 70.2 | 18.6 |  |  |
|  |  | 20 | 21.4 | 72.5 | 19.1 |  |  |
|  |  | 30 | 21.2 | 65.4 | 21.9 | — | 76.5 |
|  |  | 30 | 23.6 | 66.3 | 22.0 | — |  |
|  |  | 30 | 23.3 | 66.1 | 21.4 | — |  |
|  |  | 30 | 22.7 | 65.9 | 21.5 | — |  |
|  |  | $1^m$ | 24.2 | 58.8 | 25.0 | — | — |
|  |  | 1 | 29.1 | 57.3 | 25.9 | — | — |
|  |  | 1 | 26.8 | 62.0 | 25.4 | — | — |
| 48 | Boss. 420 | $45^s$ | 423.6 | 377.4 | 17.5 | 424.0 | 379.1 |
|  |  | 45 | 24.6 | 80.2 | 16.8 |  |  |
|  |  | 90 | 25.3 | 76.8 | 20.4 | 23.6 | 79.8 |
|  |  | 90 | 25.9 | 74.4 | 19.6 |  |  |
|  |  | $3^m$ | 30.6 | 69.5 | 23.6 | — | — |
|  |  | 3 | 29.6 | 71.4 | 23.8 | — | — |
|  |  | 6 | 35.8 | 66.9 | 27.2 | — | — |
|  |  | 6 | 36.6 | 64.8 | 28.2 | — | — |
| 49 | Lal. 3987 | $2.5^m$ | 418.8 | 379.2: | 15.3 | 419.8 | 377.1 |
|  |  | 2.5 | 19.1 | 79.6 | 15.5 |  |  |
|  |  | 5 | 21.0 | 75.1 | 18.0 | 20.3 | 77.2 |
|  |  | 5 | 21.0 | 75.9: | 18.1 |  |  |
|  |  | 10 | 25.9 | 70.2 | 21.6 | 21.3 | 76.7 |
|  |  | 10 | 24.2 | 66.7 | 21.7 |  |  |
|  |  | 20 | 28.9 | 62.4 | 24.5 | 21.9 | — |
| 50 | $\tau$ Persei | $5^s$ | 415.6 | 377.7 | 17.0 | 415.6 | 377.7 |
|  |  | 10 | 22.1 | 70.8 | 19.1 | 20.6 | 75.9 |
|  |  | 20 | 22.3 | 66.3 | 22.7 | — | 75.2 |
|  |  | 20 | 24.4 | 59.9 | 22.3 | — |  |
|  |  | 40 | 26.3 | 53.9 | 26.5 | — | — |
|  |  | 40 | 26.3 | 58.1 | 26.0 | — | — |
| 51 | $\sigma$ Cass. | $3^s$ | 412.0 | 373.6 | 17.4 | 412.4 | 375.4 |
|  |  | 3 | 12.8 | 77.2 | 16.5 |  |  |
|  |  | 5 | 13.9 | 65.7 | 18.8 | 13.1 | 70.1 |
|  |  | 10 | 14.6 | 57.1 | 22.4 | 12.2 | 69.5 |
|  |  | 15 | 16.3 | 57.7 | 24.4 |  |  |
|  |  | 40 | 16.0 | 40.6 | 29.9 | — | — |
| 52 | H. R. 8640 | $3^s$ | 411.8 | 372.5 | 16.1 | 411.4 | 370.0 |
|  |  | 3 | 10.0 | 73.8 | 15.5 |  |  |
|  |  | 6 | 13.7 | — | 18.4 | 11.8 | — |
|  |  | 6 | 11.1 | 64.8 | 18.1 |  | 67.6 |

| Star | | Exposure | $\lambda_e$ | $\lambda_{\min}$ | $D$ | $D = 17$ | |
|---|---|---|---|---|---|---|---|
| | | | | | | $\lambda_c$ | $\lambda_{\min}$ |
| | | | $\mu\mu$ | $\mu\mu$ | | $\mu\mu$ | $\mu\mu$ |
| | | 12 | 413.9 | 357.7 | 21.2 | 411.5 | 367.8 |
| | | 12 | 13.5 | 60.5 | 21.8 | | |
| | | 24 | 15.2 | 48.7 | 25.5 | 10.6 | — |
| | | 24 | 15.0 | 54.1 | 25.4 | | — |
| 53 | $\lambda$ Peg. | $5^{s}$ | 419.7 | 380.9 | 16.0 | 420.0 | 380.5 |
| | | 5 | 19.1 | 79.8 | 16.2 | | |
| | | 10 | 20.8 | 75.5 | 19.9 | 20.9 | 81.9 |
| | | 10 | 24.2 | 74.9 | 18.9 | | |
| | | 10 | 22.1 | 76.6 | 18.4 | | |
| | | 20 | 28.7 | 65.0 | 23.6 | — | 78.7 |
| | | 20 | 26.1 | 63.1 | 22.6 | — | |
| | | 40 | 31.7 | 63.9 | 27.0 | — | — |
| | | 40 | 29.8 | 62.4 | 27.7 | — | — |
| | | 80 | 35.6 | 53.0 | 32.0 | — | — |
| 54 | Boss 5868 | $25^{s}$ | 422.7 | 383.2 | 14.4 | 422.7 | 379.6 |
| | | 25 | 22.7 | — | 15.2 | | — |
| | | 45 | 26.6 | 80.7 | 16.5 | 26.6 | 80.0 |
| | | 90 | 27.0 | — | 19.7 | 26.7 | — |
| | | $3^{m}$ | 28.9 | 70.2 | 21.8 | 26.8 | 78.2 |
| | | 3 | 28.5 | 71.4 | 22.4 | | |
| | | 4 | 31.3 | 66.1 | 23.9 | 27.2 | — |
| | | 6 | 32.8 | 65.9 | 26.5 | — | — |
| 55 | Boss 5976 | $20^{s}$ | 420.3 | 379.0 | 16.7 | 420.5 | 378.6 |
| | | 40 | 20.3 | 71.0 | 20.3 | 19.7 | 78.4 |
| | | 40 | 24.4 | 73.8 | 20.4 | | |
| | | 70 | 27.0 | 70.8 | 23.0 | 20.1 | 79.0 |
| | | 70 | 23.6 | 67.2 | 22.6 | | |
| | | $2^{m}$ | 28.7 | 61.2 | 25.8 | — | — |
| | | 2 | 26.3 | 64.2 | 25.4 | — | — |
| | | 4 | 33.9 | 60.3 | 30.0 | — | — |
| | | 6 | 37.1 | 54.5 | 31.7 | — | — |
| 56 | P Cygni | 5 | 414.1 | 371.7 | 16.9 | 414.3 | 373.3 |
| | | 5 | 13.5 | 76.2 | 15.8 | | |
| | | 10 | 15.6 | — | 19.5 | 14.5 | — |
| | | 10 | 16.1 | 68.4 | 18.7 | | 69.9 |
| | | 25 | 15.6 | 51.9 | 25.0 | 11.6 | — |
| | | 25 | 16.9 | 58.8 | 24.4 | | — |
| | | 40 | 18.2 | 54.7 | 27.0 | — | — |
| 57 | $\iota$ Cephei | $3^{s}$ | 419.9 | 380.9 | 16.6 | 420.0 | 380.4 |
| | | 5 | 19.9 | 76.8 | 19.2 | 18.5 | 79.7 |
| | | 10 | 25.1 | 72.3 | 21.9 | 21.5 | 79.9 |
| | | 10 | 26.1 | 71.4 | 22.0 | | |
| | | 16 | 28.3 | 69.3 | 24.4 | — | — |
| | | 25 | 31.9 | 61.2 | 27.6 | — | — |

| Star | | Exposure | $\lambda^c$ | $\lambda_{\min}$ | $D$ | $D = 17$ | |
|---|---|---|---|---|---|---|---|
| | | | | | | $\lambda_c$ | $\lambda_{\min}$ |
| | | | $\mu\mu$ | $\mu\mu$ | | $\mu\mu$ | $\mu\mu$ |
| | | 50 | 434.5 | 353.0 | 32.5 | — | — |
| 58 | Lal. 45755 | $5^{m}$ | 421.6 | 365.7 | 21.7 | 418.3 | 373.0 |
| 59 | 56 Cygni | $5^{s}$ | 415.0 | 379.2 | 14.8 | 417.4 | 376.4 |
| | | 10 | 14.3 | 72.5 | 15.6 | 15.8 | 69.5 |
| | | 10 | 15.2 | 68.7 | 16.4 | | |
| | | 20 | 16.5 | — | 19.8 | 15.2 | 69.4 |
| | | 20 | 17.1 | 66.9 | 19.2 | | |
| | | 40 | 14.1 | 59.6 | 22.0 | 11.8 | 69.3 |
| | | 60 | 20.8 | 55.1 | 24.9 | 15.7 | — |
| 58 | Lal. 45755 | $1^{m}$ | 416.5 | — | 12.3 | 419.5 | — |
| | | 2 | 17.3 | 377.0 | 14.7 | 18.3 | 373.0 |
| | | 3 | 15.6 | 75.5 | 16.3 | 16.4 | 74.2 |
| | | 4 | 16.9 | 73.6 | 17.3 | | |
| 57 | ι Cephei | 3 | 419.9 | 385.0 | 14.2 | 419.4 | 379.9 |
| | | 5 | 16.9 | 82.2 | 15.9 | | |
| | | 5 | 19.3 | 82.0 | 13.8 | | |
| | | 10 | 19.1 | — | 18.8 | 19.0 | — |
| | | 10 | 20.3 | 76.8 | 17.7 | | 77.8 |
| | | 20 | 23.1 | 74.0 | 22.0 | 19.0 | 81.2 |
| | | 40 | 27.0 | 64.4 | 25.5 | — | — |
| | | 60 | 31.1 | 55.8 | 28.5 | — | — |
| 60 | ε Cephei | $2^{s}$ | 411.5 | 380.2 | 13.4 | 415.6 | 374.8 |
| | | 4 | 14.6 | 75.3 | 16.0 | 15.6 | 74.1 |
| | | 4 | 14.6 | — | 16.0 | | — |
| | | 8 | 17.3 | — | 19.2 | 15.9 | — |
| | | $16^{s}$ | 17.1 | 61.8 | 21.5 | 14.9 | 70.3 |
| | | 30 | 19.3 | 55.8 | 23.7 | 14.6 | — |
| | | 30 | 17.6 | 57.9 | 24.5 | | — |
| | | $1^{m}$ | 22.1 | 53.9 | 27.9 | — | — |
| 61 | A. Oe. 25685 | $4^{m}$ | 421.0 | 367.2 | 18.8 | 420.2 | 371.0 |
| 62 | Lal. 3128 | $10^{s}$ | 418.8 | 377.0 | 15.8 | 419.3 | 374.8 |
| | | 30 | 24.0 | 68.9 | 19.9 | 19.9 | 75.2 |
| | | 30 | 20.3 | 67.6 | 20.7 | | |
| | | 30 | 22.1 | 66.7 | 20.7 | | |
| | | 45 | 23.3 | 66.7 | 22.3 | 19.9 | 76.8 |
| | | 45 | 24.6 | 62.7 | 22.7 | | |
| | | $1^{m}$ | 25.3 | 58.8 | 23.3 | 20.4 | 72.8 |
| 63 | 6 Persei | $15^{s}$ | 419.7 | 373.4: | 17.1 | 420.4 | 376.2 |
| | | 15 | 20.8 | 76.4 | 16.8 | | |
| | | 30 | 23.3 | 69.5 | 19.3 | 19.9 | 78.7 |
| | | 30 | 20.8 | 74.0 | 20.7 | | |
| | | 30 | 23.8 | 70.4 | 20.7 | | |
| | | $1^{m}$ | 23.6 | 64.2 | 23.5 | — | — |
| | | 1 | 24.8 | 64.4 | 24.4 | — | — |
| | | 2 | 26.3 | 61.4 | 26.4 | — | — |

The values found for $\lambda_c$ and $\lambda_{\min}$ from the measurements were plotted in diagrams against the diameter $D$, whereby six distinct types of the resulting curves were discerned for both wave-lengths; for $\lambda_c$ the first type has been divided into two, I a, I b. The types correspond to certain intervals of spectral type and absolute magnitude in the following way:

| I | II | III | IV | | V | VI |
|---|---|---|---|---|---|---|
| K 2–Mc | K 2–Mb | F 9–K 1 | F 5–K 1 | F 1–F 4 | A | B |
| Giants | Dwarfs | Giants | Dwarfs | Giants | | |

In some cases the appearance of the curves for a certain star agrees better with the curves of an adjacent type than with the

Table 10.

| Type | Stars | $D$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 11 | 13 | 15 | 17 | 19 | 21 | 23 | 25 |
| | | $\mu\mu$ | $\mu\mu$ | $\mu\mu$ | $\mu\mu$ | $\mu\mu$ | $\mu\mu$ | $\mu\mu$ | $\mu\mu$ |
| I a | 4, 5, 7, 8, 27, 35, 42, 44. 45, 54 ......... | — | 428.0 | 428.0 | 428.0 | 428.1 | 429.0 | 430.9 | 433.8 |
| I b | 10, 12, 22, 48, 57 | — | 22.9 | 23.5 | 24.0 | 25.2 | 27.0 | 29.6 | — |
| II | 1, 2, 6, 26, 32, 33, 37, 49, 55........... | 422.8 | 22.6 | 23.1 | 24.0 | 25.5 | 27.4 | 29.4 | 31.5 |
| III | 9, 16, 19, 30, 31, 34, 40, 47, 50, 53, 63 | 18.9 | 18.4 | 18.8 | 20.0 | 21.4 | 23.6 | — | — |
| IV | 3, 11, 13, 14, 15, 17, 18, 25, 28, 29, 36, 38, 39, 41, 46, 58. 62 .................... | 13.9 | 15.6 | 17.2 | 18.0 | 18.9 | 20.8 | 22.6 | 24.5 |
| V | 23, 24, 43, 59, 50 ... | — | 10.4 | 12.9 | 15.0 | 16.4 | 17.0 | 17.8 | 20.3 |
| VI | 20, 21, 51, 52....... | — | — | 10.2 | 11.0 | 11.9 | 12.9 | 14.1 | 15.3 |

Table 11.

| Type | $D$ | | | | | | |
|---|---|---|---|---|---|---|---|
| | 11 | 13 | 15 | 17 | 19 | 21 | 23 |
| | $\mu\mu$ | $\mu\mu$ | $\mu\mu$ | $\mu\mu$ | $\mu\mu$ | $\mu\mu$ | $\mu\mu$ |
| I | — | — | 383.7 | 381.0 | 378.4 | 375.3 | 372.3 |
| II | 385.9 | 383.3 | 80.6 | 77.6 | 74.2 | 70.6 | 67.2 |
| III | — | 82.4 | 79.3 | 76.7 | 71.8 | 67.9 | 64.2 |
| IV | — | 79.9 | 77.6 | 74.0 | 69.9 | 65.3 | 60.8 |
| V | — | 77.2 | 73.5 | 71.0 | 69.1 | 63.9 | 58.9 |
| Vi | — | — | — | 70.0 | 65.4 | 62.3 | 58.3 |

curves indicated by the known spectral type of the star; in such cases the star has been referred to the adjacent type in question.

Each type of the curves was treated separately and a characteristic curve was drawn, generally nearly following one of the best determined stars but regard being paid to general tendencies of deviation shown by the other stars concerned. With the provisional curve thus determined all values of the wave-lengths within a certain interval of $D$ were reduced to normal intensity $D = 17$ by determining the residuals relative to the curve values; the agreement with the typical curves for the separate stars is very good, especially for $\lambda_{\min}$, so that we may use a large interval of $D$ for the reduction to normal intensity. The residuals were plotted against $D$ and remaining deviations of a systematic character as far as possible removed. The results are given in table 10 for $\lambda_c$ and in table 11 for $\lambda_{\min}$; they are shown graphi-

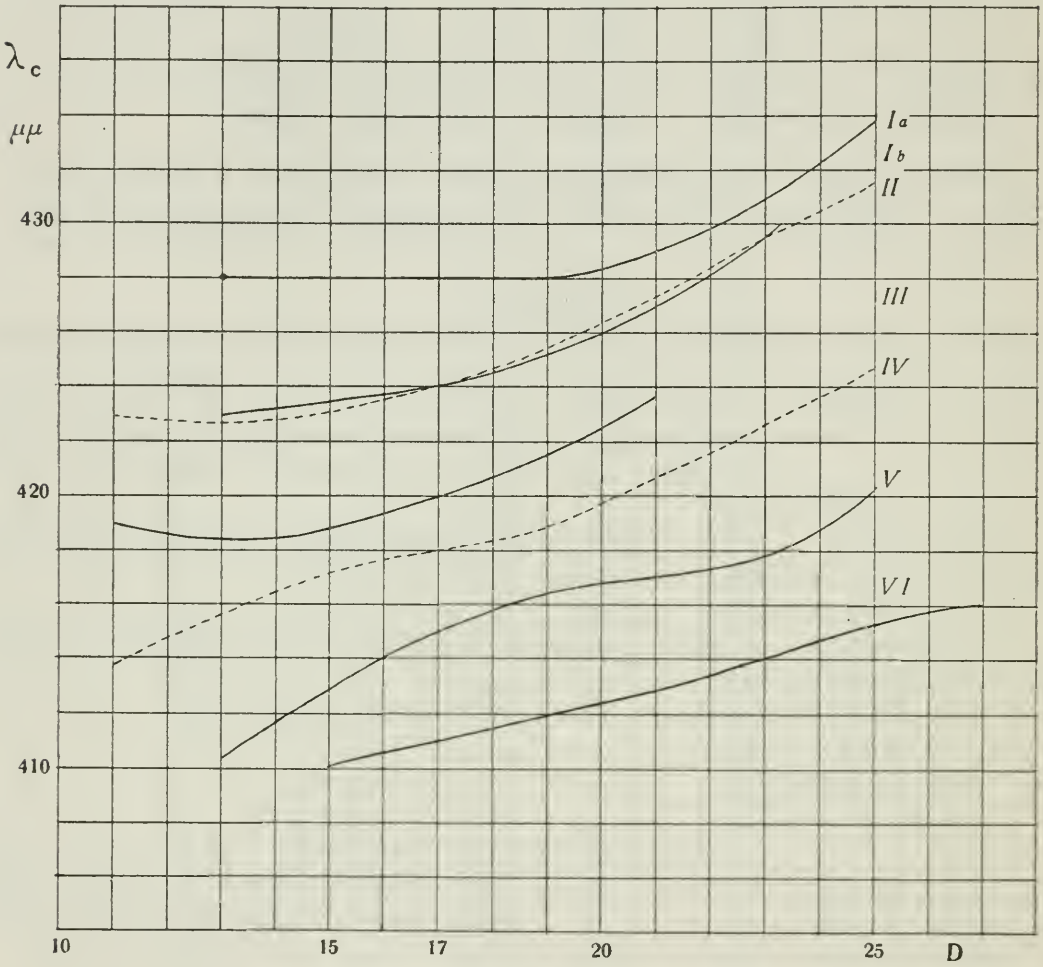

Fig. 4.

cally in fig. 4 and 5 for the interval of $D$ used in deriving the final values. The numbers of the stars in each type are those of table 9. The improved values of $\lambda_c$ and $\lambda_{\min}$ for $D = 17$ are given in the two last columns of table 9.

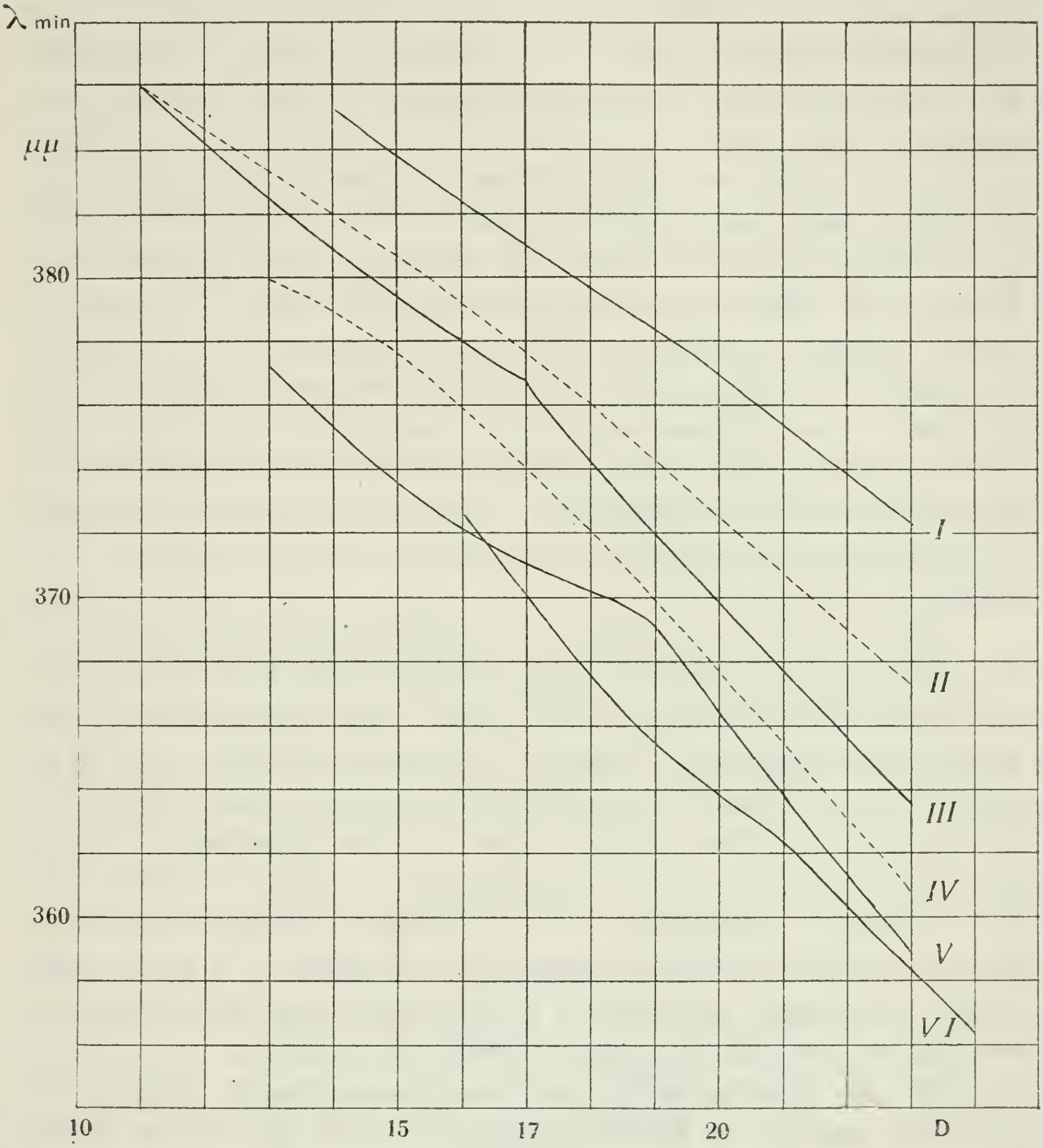

Fig. 5.

Types II and IV, in which the dwarfs predominate, are represented by dotted curves in fig. 4 and fig. 5.

The appearance of the curves shows some interesting features. For $\lambda_c$ the curves for different luminosities but nearly the same normal values of $\lambda_c$ are not very different, thus the curves I b and II are practically the same. Here there is for all types a general increase of $\lambda_c$ with the intensity of image. The differences in the appearance of the effect for the Kranz plate and the Imperial plate[1] must be ascribed to the facts mentioned about the sensibility-curves of the two plates, namely that the Imperial plate has a region of sensibility extending farther in the green region of the spectrum. The effect on $\lambda_c$ of varying intensity of image is probably not mainly due to a different gradation law for different wave-lengths, it must be connected with the fact that there exists for the plate a minimum effective intensity as well as a maximum density of the blackening. If the photographic intensity-curve has a less rapid fall in one direction than in the opposite one, the blackening will attain the maximum more rapidly on the former side with increasing intensity of the images; the contour of the perfectly blackened surface will therefore be most rapidly expanded in this direction, and thus the center of density will be removed. The displacement of $\lambda_c$ towards the red side would mean that the active photographic energy-curve generally has a perceptibly smaller inclination at the border of the strong images towards long wave-lengths than at the border of short wave-lengths.

For $\lambda_{\min}$ there is a trace of irregularity in the appearance of the effect for the giant types, at about $\lambda$ 377 $\mu\mu$ for late types and near $\lambda$ 368 $\mu\mu$ for A stars. The dwarf types on the other hand show very smooth curves. It is very probable that for the giants we are concerned with an effect of the strong ultra-violet absorption lines, for the later types the iron groups, for A stars the crowding of lines in the BALMER series towards $\lambda$ 365 $\mu\mu$ and the continuous extinction due to hydrogen extending several $\mu\mu$ beyond this limit towards shorter wave-lengths[2]. In fact a bend of the curve must occur if the intensity-curve of the stellar spectrum has a steep decrease at a certain wave-length.

For the short grating spectra corresponding to small values of $D$ the changes of the ideal intensity-curves due to the imperfectness of the optical images of the spectra will be of relatively small importance, the errors caused by the overlapping of monochromatic images may be thought to compensate each other roughly. The conditions are, however, very different for the region of the grating spectrum where the photographic intensity in a pure spectrum would be very small compared with the intensities at the strongest points of the image. The imperfectness of the optical reproduction will cause here quite a false energy-curve and the measured values of $\lambda_{\min}$ will be only apparent values. The character of the images will change when this region is attained by the border of the image; this must be the case for large values of $D$. We may express the facts very simply by stating that with increasing values of $D$ the difference of quality between the spectra and the central image vanishes. The resulting apparent photographic energy-curve of the spectra will have a relatively small inclination, and consequently the extension of the images in the direction of short wave-lengths will proceed rapidly with increasing values of $D$; we ought to have a nearly linear relation between $\lambda_{\min}$ and $D$ such as we actually find here for large values of $D$. A pure energy-curve would give a change of $\lambda_{\min}$ per unit increase of $D$ decreasing towards large values of $D$, as will be shown below. The transition between the two types of the effect may of course be a gradual one, but if there is a rapid decrease of the energy-curve in a certain region the change may be a rather abrupt one.

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[1] Cf. B. LINDBLAD, Arkiv för Mat., Astr. och Fysik, Bd 13, N:o 26, p. 24.

[2] W. HUGGINS and LADY HUGGINS, loc. cit.; W. H. WRIGHT, Lick Obs. Bulletin, Nr. 291, 1917.

Suppose $f.\cdot I$ is the photographic intensity for a certain wave-length, where the factor $f$ is due to the varying sensibility of the plate from one wave-length to another. For different exposures of the same star we then shall have at the border of the photographic image

$$h \cdot fI = C, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

where $C$ is a constant, $h$ is proportional to the relative increase of photographic intensity from one exposure to another; we may, for instance, take for $h$ the photographic intensity at the effective wave-length, which we suppose to be the same for the different images. Differentiating the logarithm of the two members we get, if we define $m = -2.5 \log h$,

$$\frac{1}{fI}\frac{d(fI)}{d\lambda} = \frac{0.4}{\log e}\frac{dm}{d\lambda}$$

A variation of $m$ is equivalent to a variation of the apparent magnitude of the star; if we use the diameter law in the form

$$m = a + b \log D,$$

we get, by putting $\frac{d(fI)}{d\lambda} = (fI)'$,

$$\frac{(fI)'}{fI} = 0.4\, b\, \frac{1}{D}\frac{dD}{d\lambda},$$

and thus

$$\frac{d\lambda}{dD} = \frac{0.4\, b \cdot fI}{D(fI)'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

The change of wave-length at the border of the image with the diameter is thus inversely proportional to the value of $D$ and the relative increase of $fI$ with the wave-length. From the curves in fig. 5 it seems as if the formula (27) were applicable to about $D = 17$, where the impurity of the spectra seems to become sensible very suddenly for G type giants.

The above arguments may now be used for an estimation of the effect of varying zenith-distance on $\lambda_{\min}$. Let $\sigma$ and $\sigma_0$ be the coefficients of extinction in the terrestrial atmosphere for the minimum and the effective wave-lengths respectively. We start from the value of $\lambda_{\min}$ for the star placed in zenith and suppose that $s$ is the increase of atmospheric mass for another position of the star. In order that the photographic intensity at the effective wave-length shall be the same in the new position we must multiply the intensity of the stellar spectrum by $e^{\sigma_0 \cdot s}$, the resulting intensities for different wave-lengths are then to be multiplied by the coefficients of transmission for the particular wave-lengths considered. Then we have the equation

$$e^{-\sigma s} \cdot fI e^{\sigma_0 s} = \frac{C}{h}$$

Putting $\sigma - \sigma_0 = \Delta\sigma$ we get for $h$ = const.

$$\Delta\sigma \frac{ds}{d\lambda} - \frac{(fI)'}{fI} = 0\,.$$

Then we get

$$\frac{d\lambda}{ds} = \Delta\sigma \frac{fI}{(fI)'},$$

and thus, combining with (27),

$$\frac{d\lambda}{ds} = \frac{\Delta\sigma D}{0.4\, b}\frac{d\lambda}{dD} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

According to the results of L. V. King mentioned in chapter I we need only consider the selective absorption due to the density changes of the air molecules and not the influence of the atmospheric dust. For 760 mm. pressure we then have

$$\sigma = 0.010\,\lambda^{-4} + \text{const.}$$

Assuming for $\lambda_c$ the value $0.420\,\mu$, for $\lambda_{\min}$ $0.375\,\mu$ we obtain $\varDelta\sigma = 0.184$.

From ten stars in the Pleiades, ranging between the magnitudes $+3.41$ and $+8.36$ I have found $b = -10.6$. Putting $D = 17$ and with the values of $\frac{d\lambda}{dD}$ for $D = 17$ taken from fig. 5 I have found the values of $\frac{d\lambda}{ds} = \frac{\varDelta\lambda_{\min}}{\varDelta_{\text{atm.}}}$ in the third column of table 12.

In deriving the values of $\frac{\varDelta\lambda_c}{\varDelta_{\text{atm.}}}$ I have started from the results in a previous investigation[1]. The values of the coefficient found for an Imperial plate for certain values of $\lambda_c$ were assumed to be valid for the same wave-lengths on the Kranz plate except for a reduction by multiplication by $\frac{5}{7}$; this factor is taken to account for the different increase of $\lambda_c$ with the spectral type for the two plates. The results are given in the second column of table 12 for each of the six types of the tables 10 and 11; the close coincidence between the zenith-corrections for $\lambda_c$ and $\lambda_{\min}$ is remarkable. For a certain zenith-distance $z$ we have

$$\varDelta_{\text{atm}} = \sec z - 1\,.$$

Table 12.

| Type | $\frac{\varDelta\lambda_c}{\varDelta_{\text{atm.}}}$ | $\frac{\varDelta\lambda_{\min}}{\varDelta_{\text{atm.}}}$ |
|---|---|---|
| | $\mu\mu$ | $\mu\mu$ |
| I | +1.1 | +1.0 |
| II | 1.2 | 1.2 |
| III | 1.4 | 1.0 |
| IV | 1.5 | 1.5 |
| V | 1.8 | 0.7 |
| IV | 2.2 | 1.9 |

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[1] B. Lindblad, loc. cit., p. 34.

In order to get the final values of $\lambda_c$ and $\lambda_{\min}$ we take the mean of the reduced values in table 9. To each separate value in the table was given a weight equal to the number of images from which it has been derived. (Such original values as are marked with the sign: indicating uncertainty in the measurements have generally not been used.) The final values of $\lambda_c$ and $\lambda_{\min}$ were then found by applying a zenith-correction in accordance with table 12. The values of the correction to zenith are undoubtedly subject to some uncertainty; it would be very valuable to verify the correction for $\lambda_{\min}$ by observation, but this is made very hard by the bad quality of the images at great zenith-distances. Generally, however, the zenith-distances for the stars here employed are rather small and therefore no appreciable uncertainty in our results due to an erroneus zenith-correction is to be feared. The final values of $\lambda_c$ and $\lambda_{\min}$ are given in table 13. The spectral type and the absolute magnitude are generally taken from ADAMS and JOY's catalogue (spectrum estimated); for early type stars the spectra are taken from Harvard observations, absolute magnitudes from WALKEY's[1] compilation of stellar parallaxes. The apparent magnitudes and the coordinates are generally given according to ADAMS and JOY's table. The columns headed $c_1$ and $c_2$ contain the corrections of $\lambda_c$ and $\lambda_{\min}$ to zenith, $n_1$ and $n_2$ are the total number of images from which the wave-lengths have been derived.

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[1] See note on p. 8.

## Table 13.

|  | Star | $m$ | R. A. 1900.0 |  | Decl. 1900.0 |  | Spectrum | Abs. mag. | $c_1$ | $c_2$ | $\lambda_c$ | $n_1$ | $\lambda_{\min}$ | $n_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  |  |  |  |  |  |  |  |  | $\mu\mu$ | $\mu\mu$ | $\mu\mu$ |  | $\mu\mu$ |  |
| 47 | ε Andr. | + 4.5 | $0^h$ | $33^m.3$ | $+28^0$ | 46′ | G 3 | + 1.3 | — 0.3 | — 0.2 | 420.2 | 9 | 375.6 | 12 |
| 62 | Lal. 3128 | 5.3 | 1 | 37.1 | 19 | 47 | K 1 | + 5.5 | — 0.5 | — 0.5 | 419.4 | 7 | 374.8 | 7 |
| 48 | Boss 420 | 5.6 |  | 47.3 | 40 | 14 | K 2 | + 0.6 | — 0.1 | — 0.1 | 423.7 | 4 | 379.4 | 4 |
| 63 | 6 Persei | 5.4 | 2 | 7.0 | 50 | 36 | G 6 | — 0.3 | — 0 0 | — 0.0 | 420.1 | 5 | 378.1 | 4 |
| 49 | Lal. 3987 | 7.8 |  | 7.5 | 67 | 13 | K 2 | + 6.0 | — 0.0 | — 0.0 | 420.9 | 7 | 377.0 | 6 |
| 50 | τ Persei | 4.1 |  | 47.2 | 52 | 21 | G 1 | + 1.5 | — 0.0 | — 0.0 | 418.1 | 2 | 376.0 | 4 |
| 44 | ϱ Persei | 3.4 — 4.2 |  | 58 9 | 38 | 27 | Mb | — 0.5 | — 0.1 | — 0.1 | 427.7 | 7 | 383.8 | 7 |
| 21 | e Plei. | 4.4 | 3 | 39.3 | 24 | 10 | B 5 |  | — 1.0 | — 0.9 | 409.9 | 6 | 367.4 | 4 |
| 20 | c Plei. | 4.0 |  | 39.9 | 24 | 4 | B 5 |  | — 1.0 | — 0.9 | 410.1 | 5 | 368.9 | 4 |
| 19 | **θ′** Tauri | 4 0 | 4 | 22.9 | 15 | 44 | G 9 | + 0.9 | — 0.8 | — 0.6 | 420.0 | 8 | 376.1 | 7 |
| 1 | Lal. 18115[1] | 7.9 | 9 | 7.6 | 53 | 7 | K 8 | + 8.6 | — 0.0 | — 0.0 | 424.0 | 3 | 376.4 | 2 |
| 2 | Lal. 18115[2] | 7.9 |  | 7.6 | 53 | 7 | K 8 | + 9.2 | — 0.0 | — 0.0 | 425.1 | 4 | 377.6 | 2 |
| 3 | 83 Cancri | 6.6 |  | 13.4 | 18 | 8 | F 6 | + 4.9 | — 0.6 | — 0.6 | 415.5 | 4 | 373.7 | 2 |
| 11 | o Leonis | 3.8 |  | 35.8 | 10 | 21 | F 4 p | + 1.3 | — 0.8 | — 0.8 | 417.7 | 6 | 372.2 | 4 |
| 12 | Boss 2603 | 6.4 |  | 35 9 | 26 | 22 | K 1 | + 0.6 | — 0.2 | — 0.2 | 424.1 | 3 | 378.4 | 3 |
| 18 | Groom. 1596 | 8.2 |  | 54.9 | 56 | 5 | G 5 | + 5.6 | — 0.2 | — 0.2 | 420.1 | 2 | 372.5 | 2 |
| 23 | η Leonis | 3.6 | 10 | 1.9 | 17 | 15 | Ap, VIIc |  | — 1.1 | — 0.4 | 412.8 | 13 | 370.7 | 7 |
| 13 | Lal. 19821 | 8.5 |  | 6.3 | 24 | 15 | G 2 | + 5.0 | — 0.4 | — 0.4 | 418.1 | 2 | 373.2 | 1 |
| 14 | 39 Leonis | 5.8 |  | 11.7 | 23 | 36 | F 5 | + 4 6 | — 0.4 | — 0.4 | 416.6 | 5 | 374.7 | 4 |
| 4 | μ Urs. Maj. | 3.2 |  | 16.4 | 42 | 0 | K 5 | + 1.8 | — 0.1 | — 0.1 | 429.7 | 8 | 382.5 | 5 |

| | Star | m | R. A. 1900.0 | | Ded. 1900.0 | | Spectrum | Abs. mag. | $c_1$ | $c_2$ | $\lambda_c$ | $n_1$ | $\lambda_{\min}$ | $n_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | μμ | μμ | μμ | | μμ | |
| 17 | Groom. 1646 | 6.5 | | 21.9 | 49 | 19 | G 1 | + 6.1 | — 0.1 | — 0.1 | 419.8 | 5 | 373.6 | 4 |
| 16 | 46 Leo. Min. | 3.7 | | 47.7 | 34 | 45 | K 1 | + 1.1 | — 0.2 | — 0.1 | 420.6 | 6 | 375.9 | 7 |
| 5 | Boss 2921 | 6.2 | | 54.0 | 36 | 38 | Ma | + 3.6 | — 0.1 | — 0.1 | 430.0 | 3 | 381.7 | 2 |
| 6 | Lal. 21185 | 7.6 | | 57.9 | 36 | 38 | Ma | + 10.8 | — 0.1 | — 0.1 | 423.5 | 2 | 378 1 | 2 |
| 15 | χ Leonis | 4.7 | | 59.9 | 7 | 53 | F 4 | + 0.1 | — 0.9 | — 0.9 | 416.2 | 8 | 372.3 | 6 |
| 7 | Boss 3089 | 4 2 | 11 | 40.7 | 7 | 5 | Ma | + 1.9 | — 0.7 | — 0.7 | 427.7 | 6 | 382.8 | 4 |
| 8 | δ Virg. | 3.7 | 12 | 50.6 | 3 | 56 | Mb | + 1.3 | — 0.9 | — 0.8 | 426.3 | 4 | 379.6 | 4 |
| 9 | ε Virg. | 3.0 | | 57.2 | 11 | 30 | G 6 | + 0.4 | — 0.7 | — 0.5 | 419.5 | 4 | 376.7 | 1 |
| 10 | Boss 3398 | 5.9 | 13 | 1.5 | 23 | 9 | Mc | + 1.8 | — 0.3 | — 0.3 | 423.8 | 2 | 378.4 | 2 |
| 25 | β Comæ | 4.3 | | 7.2 | 28 | 23 | G 1 | + 4.6 | — 0.3 | — 0.3 | 418.1 | 8 | 372.7 | 8 |
| 26 | Lal. 24652 | 7.0 | | 11.9 | 17 | 35 | K 3 | + 6.7 | — 0.4 | — 0.4 | 422.4 | 6 | 377.7 | 6 |
| 24 | Lal. 25224 | 5.5 | | 34.7 | 11 | 15 | A | + 7.6 | — 0.9 | — 0.4 | 416.1 | 6 | 369.3 | 6 |
| 27 | Boss 3605 | 6.0 | | 53.8 | 15 | 8 | K 4 | + 1.1 | — 0.5 | — 0.4 | 429.3 | 2 | 383.3 | 2 |
| 22 | ϱ Bootis | 3.8 | 14 | 27.5 | 30 | 49 | K 3 | + 0.9 | — 0.2 | — 0.2 | 424.2 | 3 | 383.2 | 2 |
| 35 | 109 Herc. | 3.9 | 18 | 19.4 | 21 | 43 | K 2 | + 1.0 | — 0.5 | — 0.4 | 427.0 | 7 | 378.6 | 4 |
| 28 | σ Drac. | 4.8 | 19 | 32.6 | 69 | 29 | G 8 | + 5.4 | — 0.0 | — 0.0 | 420.0 | 7 | 374.2 | 5 |
| 29 | Lal. 38287 | 7.2 | | 58.0 | 15 | 20 | G 8 | + 6.6 | — 0.7 | — 0.7 | 419.1 | 5 | 373.9 | 4 |
| 37 | Lal. 38683 | 7.3 | 20 | 6.6 | 15 | 53 | K 1 | + 6.7 | — 0.5 | — 0.5 | 421.6 | 4 | 376.7 | 3 |
| 30 | 24 Vulpec. | 5.4 | | 12.5 | 24 | 22 | G 7 | + 0.7 | — 0.4 | — 0.3 | 421.1 | 7 | 376.0 | 7 |
| 56 | P Cygni | 4.9 | | 14.1 | 37 | 43 | B 4 p | (− 2.7) | — 0.5 | — 0.4 | 413.0 | 6 | 371.8 | 3 |
| 38 | Groom. 3150 | 6.1 | | 16.5 | 66 | 32 | F 7 | + 6.1 | — 0.1 | — 0.1 | 417.9 | 9 | 374.1 | 4 |
| 31 | γ Cygni | 2.3 | | 18.6 | 39 | 56 | F9p,XIIIc | + 1.3 | — 0.1 | — 0.1 | 418.5 | 5 | 378.3 | 2 |
| 36 | β Delph. | 3,7 | | 32.9 | 14 | 15 | F 3 | + 1.6 | — 0.6 | — 0.6 | 415.7 | 4 | 374.0 | 3 |
| 59 | 56 Cygni | 5.1 | | 46.6 | 43 | 41 | A 5 | (− 1.4) | − 0.2 | — 0.1 | 415.1 | 7 | 370.7 | 5 |
| 32 | $61^1$ Cygni | 5.6 | 21 | 2.4 | 38 | 15 | K 7 | + 7.9 | — 0.1 | — 0.1 | 425.0 | 10 | 377.0 | 9 |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 33 | $61^2$ Cygni | 6.3 | | 2.4 | 38 | 15 | K 8 | + 8.7 | — 0.1 | — 0.1 | 425.1 | 11 | 375.9 | 7 |
| 34 | ζ Cygni | 3.4 | | 8.7 | 29 | 49 | G 6 | — 0.4 | — 0.3 | — 0.2 | 421.3 | 8 | 377.4 | 7 |
| 39 | τ Cygni | 3.8 | | 10.8 | 37 | 37 | F 1 | + 1.9 | — 0.2 | — 0.2 | 416.7 | 8 | 372.0 | 4 |
| 41 | A. G. Ber. 8366 | 8.8 | | 39.7 | 24 | 53 | K 1 | + 6.3 | — 0.4 | — 0.4 | 419.3 | 2 | 374.2 | 2 |
| 43 | ϰ Peg. | 4.3 | | 40.1 | 25 | 11 | F 2 | + 3.4 | — 0.6 | — 0.2 | 414.4 | 5 | 372.1 | 4 |
| 42 | Boss 5602 | 6.5 | | 41.8 | 25 | 6 | K 3 | + 1.5 | — 0.3 | — 0.3 | 426.4 | 3 | 380.1 | 3 |
| 60 | ε Cephei | 4.2 | 22 | 11.3 | 56 | 33 | A 5 | + 3.8 | — 0.1 | — 0.0 | 415.2 | 7 | 373.1 | 3 |
| 64 | Krüger 60 | 9.2 | | 24.5 | 57 | 12 | Mb | + 11.2 | — 0.2 | — 0.2 | 424.2 | 2 | 375.2 | 2 |
| 52 | H. R. 8640 | 5.2 | | 37.0 | 39 | 43 | B 2 | | — 0.2 | — 0.2 | 411.1 | 8 | 368.4 | 5 |
| 40 | η Peg. | 3.1 | | 38.3 | 29 | 42 | G 2 | + 1.2 | — 0.2 | — 0.2 | 420.3 | 4 | 376.8 | 3 |
| 54 | Boss 5868 | 6.1 | | 39.6 | 38 | 56 | K 4 | + 0.9 | — 0.1 | — 0.1 | 425.5 | 7 | 378.9 | 4 |
| 53 | λ Peg. | 4.1 | | 41.7 | 23 | 2 | G 7 | — 0.5 | — 0.4 | — 0.3 | 420.1 | 5 | 380.3 | 7 |
| 57 | ι Cephei | 3.7 | | 46.1 | 65 | 40 | K 0 | + 1.1 | — 0.0 | — 0.0 | 419.8 | 10 | 379.8 | 9 |
| 55 | Boss 5976 | 5.6 | 23 | 8.5 | 56 | 37 | K 5 | + 6.6 | — 0.0 | — 0.0 | 420.0 | 5 | 378.7 | 5 |
| 58 | Lal. 45755 | 7.6 | | 16.8 | 43 | 33 | K 0 | + 6.3 | — 0.1 | — 0.1 | 417.7 | 5 | 373.5 | 4 |
| 61 | A. Oe. 25685 | 7.5 | | 26.5 | 58 | 37 | K 0 | + 5.8 | — 0.0 | — 0.0 | 420.2 | 1 | 371.0 | 1 |
| 51 | σ Cass. | 4.9 | | 53.9 | 55 | 12 | B 2 | | — 0.1 | — 0.1 | 412.4 | 5 | 371.9 | 5 |
| 45 | B. D. + $26^0$4721 | 7.8 | | 54.2 | 26 | 43 | K 2 | + 0.1 | — 0.2 | — 0.2 | 438.2 | 4 | 385.0 | 1 |
| 46 | 85 Peg. | 5.9 | | 56.9 | 26 | 33 | G 1 | + 5.8 | — 0.4 | — 0.4 | 417.4 | 7 | 371.8 | 6 |

The mean errors of $\lambda_c$ and $\lambda_{\min}$ computed from the differences from the mean values of each star would have rather small values. However, if we take into account possible deviations of a more systematic kind from one plate to another, due for instance to a possible difference in the quality of the plates, unsteadiness of the air, or a somewhat varying personal equation in the measurements (especially for $\lambda_{\min}$) we must probably admit possible errors in the final results of the order of magnitude 0.5 — 1 $\mu\mu$. Three stars, $\theta^1$ Tauri, Lal. 45755, $\iota$ Cephei, have been measured here on more than one plate; for these stars the values from the two plates agree well. I have sought especially to eliminate systematic errors in the magnitude of the luminosity-effects by taking stars of different luminosities but nearly the same spectral type on the same plate. The extreme redness found here for the star B. D. 26°4721 is interesting but ought to be tested with new plates. In fig. 6 and fig. 9 this star is not to be found because it falls outside the margins of the figures. The values for the star A. Oe. 25685 which only depend on one measured image are not reproduced in the figures.

The results contained in the above table are shown graphically in fig. 6—9. Let us consider first fig. 6, giving the effective wave-length $\lambda_c$ as a function of the spectral type. For late type stars the open squares denote giants with absolute magnitude (apparent mag. at the distance 10 parsec) $M < +0.3$, the open circles giants nor far from $M = +1.0$; the black cirkles represent dwarfs with $M < +7.0$, the black squares represent very small dwarfs, $M > +7.0$. For early type stars, the B and A stars, I have used open circles, except for the two dwarfs $\varepsilon$ Cephei and Lal. 25224. The absolute magnitudes are perhaps to be estimated considerably smaller than + 1.0. The star P Cygni of a peculiar spectral type has not been used in the first two diagrams.

The general character of the diagram is the same as in the corresponding diagrams of previous papers, but some characteristic features are unusually pronounced here, especially the constancy of the effective wave-length in the giant series between G 2 and K 0 and the very strong increase between types K 1 and K 5.

In this connection we are most interested in the luminosity-effect. Because of the smaller general variation of $\lambda_c$ on Kranz plates compared with the plates used in previous investigations

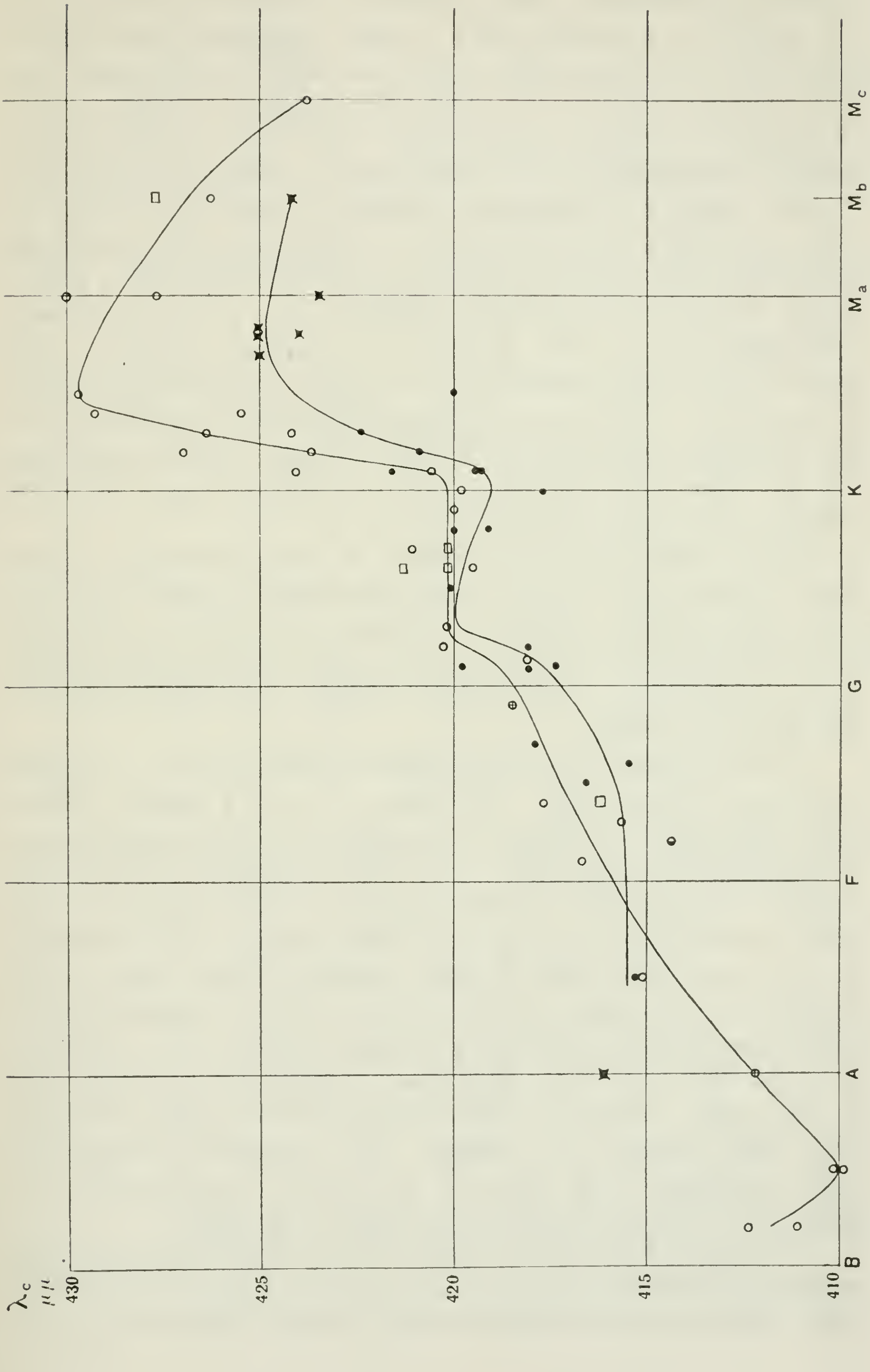

Fig. 6.

we must expect the luminosity-effect to be smaller in a corresponding proportion. But in addition it seems as if for this sort of plates the luminosity-effect vanishes altogether about spectral type G 5. In the preceding chapter we arrived at the result that the luminosity-effect in the effective wave-length is mainly due to a change of effective temperature. Now it is evident that the effective wave-length on the Kranz plate is very slightly sensitive to the change of temperature between the types G 2 and K 0, so that a vanishing of the difference in $\lambda_c$ between giants and dwarfs about type G 5 must be expected. On the other hand a glance at fig. 7 shows us immediately that there is a considerable luminosity-effect in $\lambda_{\text{min}}$ for the type in question, though $\lambda_{\text{min}}$ also proves to be nearly insensible to a change of temperature between G 2 and K 0. For $\lambda_{\text{min}}$ the luminosity-effect must therefore be of an essentially different and a more strongly selective kind than for $\lambda_c$. Thus the results derived here strongly support our main results in the preceding discussion.

Fig. 7 gives $\lambda_{\text{min}}$ as a function of the spectral type. It is seen at once that the two color-equivalents $\lambda_c$ and $\lambda_{\text{min}}$ have some characteristic features in common, in addition to the constancy between G 2 and K 0 and the rapid increase between K 1 and K 5 there are especially the changes of the giant curves at the types B 5 and K 5.

The relatively very strong luminosity-effect for $\lambda_{\text{min}}$ is caused by several circumstances. The effect in $\lambda_{\text{min}}$ of a change of effective temperature is reduced to a minimum by the absorption in the objective lenses causing a very steep inclination of the photographic intensity-curve at the boundary of the images towards short wave-lengths. At the same time, however, the absorption of the lenses has caused a sharp boundary of the images to fall exactly in the region of the extremely selective luminosity-effect due to some wide ultra-violet groupings of lines. For a reflector or a refractor with U.-V. glass these circumstances will not occur to the same extent; for these instruments the method followed here must certainly be modified. It is probable, however, that such instruments will prove to be superior in several other respects for a rigorous study of the effects in question.

The stars between G 1 and G 9 may be divided into three groups with different luminosities. The mean values of $\lambda_{\text{min}}$ and $M$ with their mean errors are given below for each of the three groups.

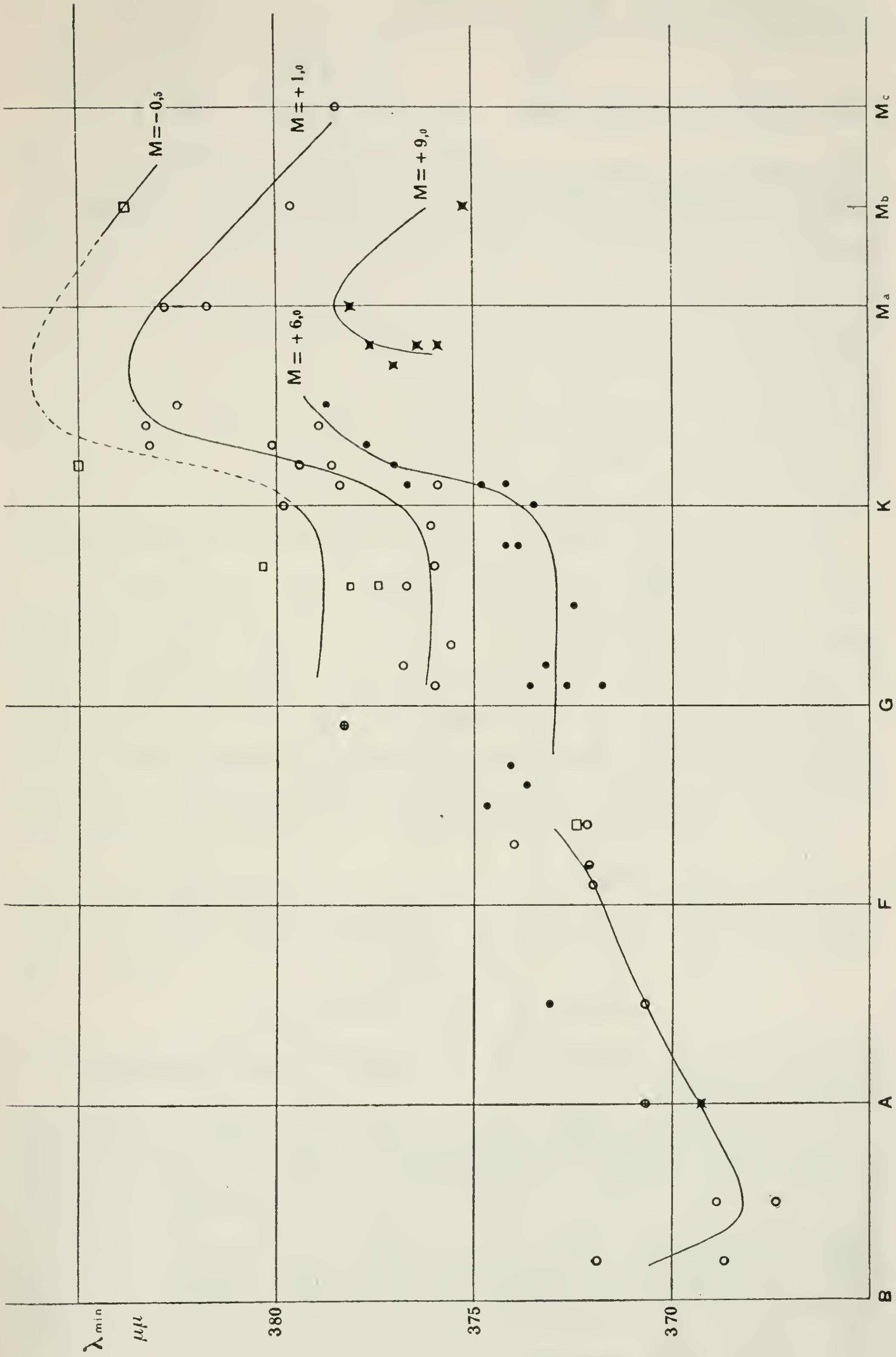

Fig. 7.

| Number of stars | Spectral type | $M$ | $\lambda_{\min}$<br>$\mu\mu$ |
|---|---|---|---|
| 3 | G 6 | $-0.4 \pm 0.0$ | $378.6 \pm 0.9$ |
| 6 | G 5 | $+1.0 \pm 0.2$ | $376.2 \pm 0.2$ |
| 7 | G 4 | $+5.6 \pm 0.8$ | $373.1 \pm 0.3$ |

The results are illustrated in fig. 8. The change of $\lambda_{\min}$ per unit absolute magnitude increases with decreasing $M$; this result is analogous to that found by F. H. Seares[1] for the luminosity-effect in the color-index.

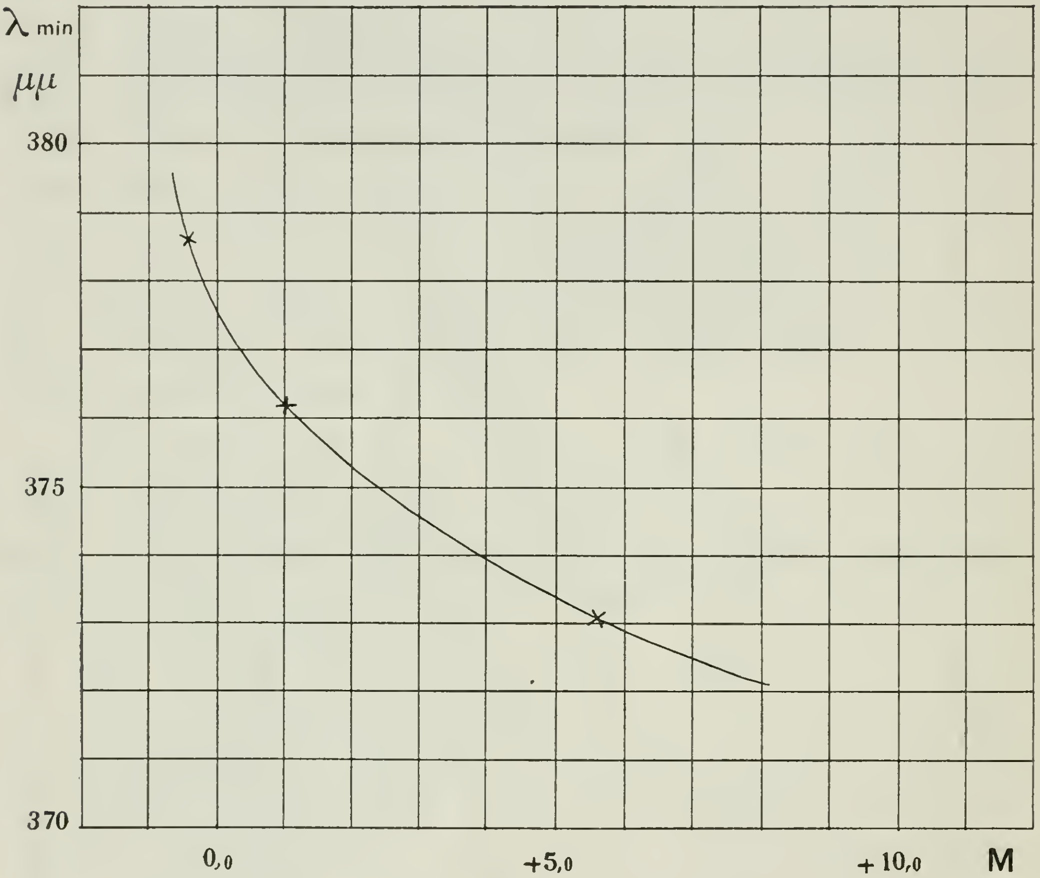

Fig. 8.

The luminosity-effect in $\lambda_{\min}$ seems to be fairly constant for all later types; paying some attention to fig. 8 I have been able to draw in fig. 7 nearly parallel curves corresponding to the absolute magnitudes $-0.5$, $+1.0$, $+6.0$, $+9.0$. The curve for $M = +6$ is perhaps drawn somewhat too high at type K 5. The luminosity-effect in $\lambda_{\min}$ seems to be relatively large for the types

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[1] loc. cit. in the introduction.

K 5 — K 8, probably because the general luminosity-effect is especially well developed here as shown in fig. 6. The resulting effect for $\lambda_{\min}$ wil therefore be due to a composition of two tendencies working in the same direction.

We cannot draw any definite conclusions here about the luminosity-effects for early type stars.

We now turn to discuss the extremely important question how far the absolute magnitudes may be determined without a knowledge of the spectral types of the stars, only by the use of the two color-equivalents $\lambda_c$ and $\lambda_{\min}$. In fig. 7 it is seen that between K 0 and K 5 we must have a very rigorous determination of the spectral type in order to get the absolute magnitudes with any certainty. But at the same time we see from fig. 6 that rather good values of the spectral types may be secured from $\lambda_c$ just for the spectral types of this interval, because there is here a very strong increase of $\lambda_c$ with increasing spectral type. An elimination of the spectral type thus seems quite possible.

In fig. 9 the two color-equivalents are plotted against each other; the scale for $\lambda_{\min}$ is half that for $\lambda_c$. I have drawn in the diagram dotted lines to discriminate roughly between giants and dwarfs. It is evident that there exist three distinct groups of giants between which large discontinuities of the wave-lengths occur; it is possible that this fact may be of importance for the problem of stellar evolution. The limiting values of $\lambda_c$ and $\lambda_{\min}$ for the three giant groups as given in fig. 9 are as follows,

| White giants | Yellow giants |
|---|---|
| $\lambda_c < 417.9\ \mu\mu$, $\lambda_{\min} < 372.8\ \mu\mu$ | $\lambda_c < 421.4\ \mu\mu$ $\lambda_{\min} > 375.3\ \mu\mu$ |

Red giants

$\lambda_c > 423.5\ \mu\mu$, $\lambda_{\min} > 378.2\ \mu\mu$

The value of the limit of $\lambda_c$ for the first group is perhaps somewhat too large for stars of regular types, the giant *o* Leonis situated near the limit is of the type F 4 p, according to Adams and Joy, and is recorded by Miss Maury as having a spectrum of composite type.

In the case of the dwarfs there is also a decided difference between »yellow» and »red» ones, but the abruptness of the change in $\lambda_{\min}$ has probably no deep signification, being due to a change at the pointed end of the dwarf spectrum. Because of the existence of the strong ultra-violet lines a change of temperature may well cause a sudden increase of $\lambda_{\min}$ when we proceed towards later types; moreover, we are here concerned with a

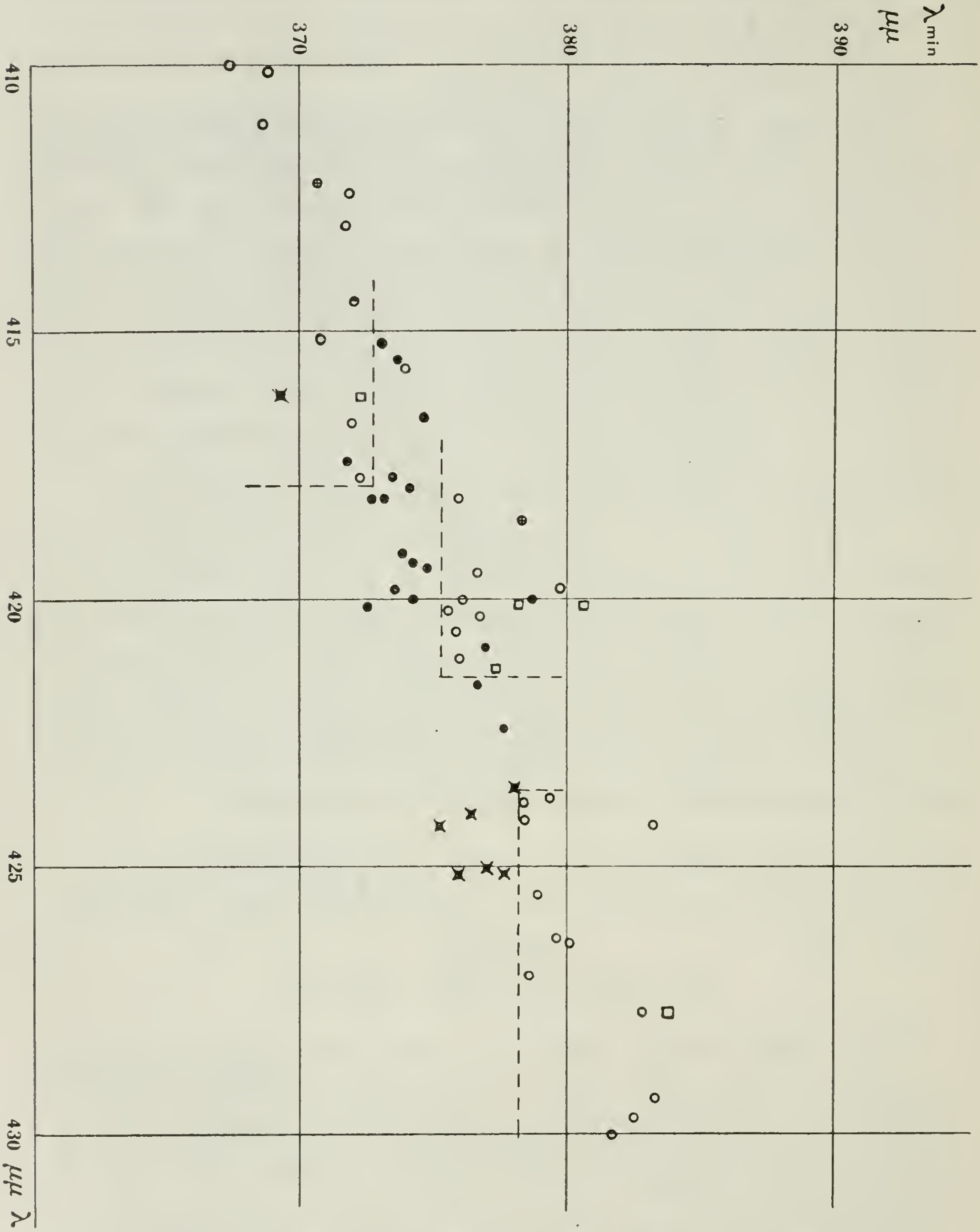

Fig. 9.

type of images for which observational errors may cause appreciable deviations. A third group of dwarfs corresponding to the B and A giants is perhaps indicated by the single A dwarf at $\lambda_c = 416.1$ $\mu\mu$, $\lambda_{\min} = 369.3$ $\mu\mu$.

The lines drawn seem to isolate pretty well giants and dwarfs between A 5 and Mb, but there is at least one region in which the values of $\lambda_c$ and $\lambda_{\min}$ for $D = 17$ seem to leave the matter undecided; it seems as if dwarfs of types K 1 — K 5 may coincide with pronounced giants of type G 5 — G8. The difference of the characteristic curves in fig. 5 is, however, so great that we can probably always discriminate between the two apparent possibilities here, if we have measured a sufficiently great number of images with different intensities.

When the general character of the luminosity has thus been determined we can derive a more accurate value of the spectral type by means of fig. 6, and get a better estimation of the absolute magnitude from fig. 7. In this way we may, at least theoretically, proceed in several successive approximations. We may also use another process drawing directly in fig. 9 curves for equal spectral types and equal luminosities. How far there exists any possibility of discriminating in such a diagram between the giants of type M and those of the type K 2 — K 5 is not clear. The early B stars can probably be distinguished from A stars by means of the characteristic curves in fig. 5. Between G 1 and K 0 the wave-lengths on a Kranz plate do not admit of a rigorous determination of the spectral type.

The conclusion drawn in the preliminary investigation referred to above have thus been confirmed in the new investigation here; we are able to state that *a discrimination between different degrees of luminosity is possible to a great extent by means of the effective and minimum wave-lengths determined from the grating spectra, without any reference to the spectral lines.* We may also state from fig. 6, as far as we are concerned with the stellar types here investigated, that *stars with* $\lambda_c > 426\ \mu\mu$ *in the Kranz system are giants.*

In this connection we may finally add some words concerning the systems of effective wave-lengths used in various investigations connected with the Zeiss-Heyde astrograph at the Observatory of Upsala. The first system, employing Lumière plates was that of Ö. Bergstrand and B. Lindblad[1]; after this investigation was performed the position of the medium lense of the objectives was somewhat changed in order to get good images on a larger part of the plate. In a following investigation[1] I used an Imperial plate, 9220 A, S. S., which was found to give very favourable results; this system has formed the basis for the investigations of K. LUNDMARK and B. LINDBLAD[2] on the effective wave-lengths of nebulæ and clusters. In the investigation just mentioned measurements for late type stars were also performed on Wellington plates, Press plate 3033. This system is reduced to Imperial by subtracting 3 $\mu\mu$. In the preliminary notice[3] on the two color-indices of the grating spectra I used Imperial S. S., 9629 A (Imperial II). The system derived here is reduced to the first Imperial system by adding 3 $\mu\mu$ to $\lambda_c$. In all the systems mentioned here except the first, the normal intensity of image determined by the diameter of the central image has been $D = 14$, expressed in hundredths of a revolution of the micrometer; in the first investigation we used $D = 15$ as normal intensity.

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[1] Arkiv för Mat., Astr. och Fysik, Bd 11, N:o 17, 1916.

To these systems with the first Imperial system (Imperial I) as a standard I have here added one on the Kranz plate, normal diameter $D = 17$, a system which differs perceptibly from the others, especially by the small amplitude of variation. The wave-lengths may be measured here with great accuracy, but in order to get a color-equivalent nearly analogous to the usual color-index the Imperial system is to be preferred. There are 11 stars common to Kranz and Imperial I; they may be fairly connected by the formula

$$\lambda_I - 410 = 1.87\,(\lambda_K - 410)\,.$$

However, 13 stars common for Kranz and Imperial II indicate a rather complicated curve for the relation in question; of course this may be due to a difference between Imperial I and Imperial II, but this is not very probable.

In what concerns $\lambda_{\min}$ the results derived in the present investigation seem to agree well with those of the preliminary investigation. The values of $\lambda_{\min}$ are systematically somewhat smaller here, this is probably due exclusively to the greater value of $D$ used as normal value and the different method of measuring;

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[1] Arkiv för Mat., Astr. och Fysik, Bd 13, N:o 26, 1918.

[2] Astron. Nachr. 205, 161, 1917; Astrophys. Journ. 46, 206. Astrophys. Journ. 50, 376, 1919.

[3] Astrophys. Journ. 49, 289, 1919.

in the preliminary notice $\lambda_{\min}$ was measured with a double thread of the micrometer, here it was obtained by means of a touching single thread.

## 2. Application of the method to a region in Cepheus.

A plate taken for the purpose of securing images of the Mb dwarf Krüger 60 turned out to be of a very good quality; for the best region of the plate accurate determinations of $\lambda_c$ and $\lambda_{\min}$ for the faint unknown stars seemed quite possible. Another plate was then secured for the same region, and all stars with favorable images situated in a certain small square (about 4 cm$^2$) on the plates were measured. The data of the plates are given below.

| Plate | Date | Sidereal time | Exposures |
|---|---|---|---|
| 1 | Sept. 29, 1919 | $0^h32^m-2^h31^m$ | $35^m, 4^m, 80^m$ |
| 2 | Oct. 24, 1919 | $2^h30.5^m-4^h45^m$ | $128^m$ |

The coordinates of the optical center of the plate are somewhat different for the two plates; through this arrangement the influence of systematic errors depending on the position of the stars relative to the plate center is diminished.

A reproduction of the second plate is given in Pl. I at the end of the paper. The square marks the region mentioned. The first star in the tables below, B. D. 57°2514, is situated somewhat outside the upper left corner of the square.

The measurements were reduced by means of the curves in fig, 4 and 5 and corrections for the zenith-distance were applied. Images with $D < 14$ were generally excluded; further generally only those stars have been given in the tables below for which at least two good determinations have been secured. Exceptions are sometimes made, especially for red giants, the spectra of which are very characteristic. The results of the measurements, and the final values of the wave-lengths are given in table 14.

Table 14.

| Star | | Expo-sure | $\lambda_c$ | $\lambda_{\min}$ | $D$ | $\lambda_c$ | $\lambda_{\min}$ |
|---|---|---|---|---|---|---|---|
| | B. D. | min. | $\mu\mu$ | $\mu\mu$ | | $\mu\mu$ | $\mu\mu$ |
| 1 | $57^0 2514$ | 128 | 417.1 | 371.0 | 16.4 | 418.0 | 371.0 |
| | | 80 | 18.6 | 74.4 | 15.9 | | |
| 2 | $56^0 2768$ | 128 | 417.1 | 368.2 | 20.2 | 413.5 | 372.9 |
| | | 80 | 15.2 | 66.7 | 20.0 | | |
| | | 35 | 11.1 | 75.7 | 15.7 | | |
| 3 | — | 128 | 418.2 | 374.7 | 14.0 | 419.6 | 370.5 |
| | | 80 | 18.6 | 75.9 | 14.4 | | |
| 4 | $57^0 2519$ | 128 | 417.6 | 370.6 | 18.4 | 416.7 | 371.4 |
| | | 80 | 18.8 | — | 19.3 | | |
| | | 35 | 15.4 | 72.7 | 15.3 | | |
| 5 | $57^0 2521$ | 128 | 430.9 | 362.0 | 26.4 | 422.6 | 379.4 |
| | | 80 | 33.6 | 64.6 | 27.0 | | |
| | | 35 | 28.3 | 70.8 | 23.0 | | |
| 6 | — | 128 | 415.8 | 372.5 | 16.0 | 415.1 | 372.4 |
| | | 80 | 12.6 | 73.8 | 17.0 | | |
| 7 | — | 128 | 417.1 | 374.2 | 16.7 | 416.3 | 374.4 |
| | | 80 | 15.4 | 77.4 | 16.1 | | |
| 8 | $56^0 2772$ | 128 | 422.9 | 382.2: | 15.6 | 422.7 | 382.9 |
| | | 80 | 22.1 | 86.5 | 15.4 | | |
| 9 | $56^0 2773$ | 128 | 416.7 | 366.1 | 19.8 | 414.2 | 369.9 |
| | | 80 | 16.5 | 61.8 | 21.4 | | |
| | | 35 | 14.1 | 69.1 | 17.6 | | |
| 10 | — | 128 | 416.3 | 370.2 | 17.7 | 414.9 | 371.9 |
| | | 80 | 15.4 | 72.3 | 18.2 | | |
| 11 | — | 128 | 415.6 | 372.3 | 16.6 | 415.2 | 372.3 |
| | | 80 | 14.6 | 73.2 | 16.9 | | |
| 12 | — | 128 | 415.2 | 368.9 | 16.7 | 414.4 | 369.8 |
| | | 80 | 13 5 | 71.2 | 17.0 | | |
| 13 | $57^0 2528$ | 128 | 415.0 | 368.7 | 18.1 | 416.6 | 371.1 |
| | | 80 | 18.0 | 68.2 | 17.4 | | |
| | | 35 | 16.1 | 78.5 | 14.2 | | |
| 14 | $57^0 2529$ | 128 | 415.4 | 370.4 | 17.6 | 413.8 | 369.7 |
| | | 80 | 13.0 | 69.1 | 16.7 | | |
| 15 | — | 128 | 415.2 | 376.6 | 15.9 | 413.6 | 375 |
| | | 80 | 11.1 | 80.7 | 15.5 | | |
| 16 | — | 128 | 414.3 | 381.3 | 14.6 | 416.6 | 378.3 |
| | | 80 | 16.5 | 82.4 | 14.3 | | |
| 17 | — | 128 | 413.3 | 378.1 | 14.3 | 412.2 | 373 |
| | | 80 | 09.4 | 80.9 | 15.1 | | |
| 18 | — | 128 | 415.2 | 376.2 | 14.7 | 416.3 | 372.1 |
| | | 80 | 12.6 | 74.4 | 14.6 | | |
| 19 | $56^0 2777$ | 128 | 426.8 | 377.9 | 18.5 | 425.3 | 378.9 |
| | | 80 | 25.0 | 73.7 | 18.3 | | |
| | | 35 | 25.2 | 85.3 | 14.3 | | |

| Star | | Exposure | $\lambda_c$ | $\lambda_{min}$ | D | $\lambda_c$ | $\lambda_{min}$ |
|---|---|---|---|---|---|---|---|
| | B. D. | min. | $\mu\mu$ | $\mu\mu$ | | $\mu\mu$ | $\mu\mu$ |
| 20 | — | 128 | 416.1 | 371.0 | 18.8 | 414.5 | 373.9 |
| | | 80 | 15.8 | 71.0 | 18.1 | | |
| | | 35 | 12.6 | 76.4 | 15.6 | | |
| 21 | $57^{0}2534$ | 128 | 417.1 | — | 17.7 | 415.2 | 372.9 |
| | | 80 | 15.0 | 373.8 | 16.9 | | |
| | | 35 | 13.3 | 75.5 | 15.3 | | |
| 22 | — | 128 | 419.5 | 378.1 | 17.0 | 417.0 | 377.3 |
| | | 80 | 14.6 | 76.6 | 17.0 | | |
| 23 | — | 128 | 417.6 | 381.3 | 14.3 | 417.8 | 378.6 |
| | | 80 | 15.4 | 84.5 | 13.7 | | |
| 24 | — | 128 | 415 2 | 375.3 | 17.3 | 415.0 | 374.7 |
| | | 80 | 15.0 | 74.2 | 16.6 | | |
| 25 | — | 128 | 416.5 | 376.0 | 16.8 | 416.0 | 374.1 |
| | | 80 | 15.4 | 74.2 | 16.3 | | |
| 26 | $56^{0}2780$ | 80 | 421.4 | 361.2 | 23.2 | 417.4 | 374.9 |
| | | 35 | 19.1 | 70.6 | 19.1 | | |
| 27 | $56^{0}2779$ | 128 | 424.8 | 383.0 | 13.7 | 425.6 | 377.3 |
| | | 80 | 24.2 | 81 7 | 13 4 | | |
| 28 | — | 128 | 416.3 | 371.0 | 18.1 | 415.1 | 373.5 |
| | | 80 | 14.8 | 72.7 | 17.6 | | |
| 29 | $56^{0}2782$ | 128 | 419.7 | 373.6 | 18.0 | 418.9 | 377.8 |
| | | 80 | 20.3 | 76.4 | 18 2 | | |
| | | 35 | 17.1 | 80.9 | 14.9 | | |
| 30 | — | 128 | 418.2 | 364.6 | 19.6 | 418.2 | 372.7 |
| | | 80 | 21.8 | 69.1 | 20.0 | | |
| 31 | — | 128 | 424.6 | 380.5 | 16.3 | 425.1 | 379.3 |
| | | 80 | 26.1: | — | 16.6 | | |
| 32 | Krüger 60 | 128 | 422.5 | 374.9 | 17.2 | 424.2 | 375.2 |
| | | 80 | 26.3 | 75.7 | 16.9 | | |
| 33 | — | 128 | 423.8 | 386.2 | 13.8 | 423.4 | 381.5 |
| 34 | $56^{0}2785$ | 80 | 440.5 | 364.6 | 29.3 | 428.0 | 381.5 |
| | | 35 | 33.0 | 71.0 | 24.5 | | |
| 35 | $57^{0}2544$ | 128 | 415.0 | 372.9 | 17.4 | 416.6 | 374.8 |
| | | 80 | 18.2 | 77.7 | 16 2 | | |
| 36 | $56^{0}2786$ | 128 | 427.8 | — | 13.2 | 426.4 | 384.0 |
| | | 80 | 25.3 | 387.5 | 14.5 | | |
| 37 | $56^{0}2787$ | 128 | 432.1 | 379.4 | 20.2 | 432.0 | 387.0 |
| | | 80 | 32.8 | 81.3 | 21.1 | | |
| | | 35 | 32.8 | 89.9 | 17.1 | | |
| 38 | $56^{0}2788$ | 128 | 429.3 | 377.7 | 20.3 | 430.3 | 383.5 |
| | | 80 | 33.4 | 81.3 | 19.8 | | |
| | | 35 | 29.1 | 84.7 | 15.8 | | |
| 39 | $56^{0}2789$ | 128 | 418.4 | 361.4 | 23.0 | 416.2 | 374.7 |
| | | 80 | 22.1 | 65.2 | 22.4 | | |
| | | 35 | 17.3 | 70.2 | 18.3 | | |

The values of $\lambda_c$ and $\lambda_{\min}$ are plotted in fig. 10, in which the lines of fig. 9 have been drawn (with two slight modifications).

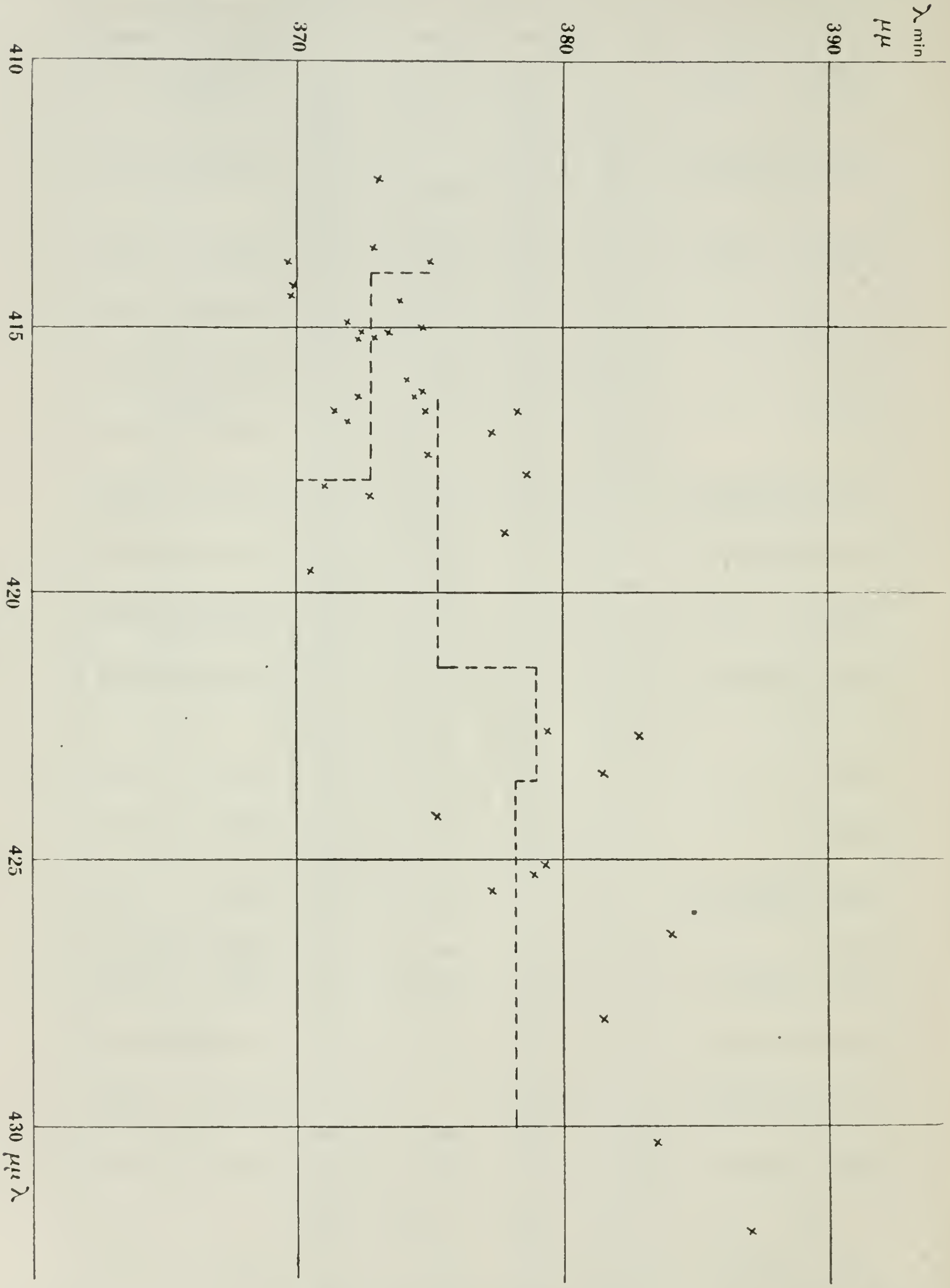

Fig. 10.

As far as possible spectral types and absolute magnitudes were then determined by means of fig. 6 and 7; in the estimation of $M$ I have not gone outside the limiting values found in Adams and Joy's catalogue.

The visual apparent magnitudes ($m$) of the stars were derived in the following way. A sequence of photographic magnitudes was found by means of the values of $D$ for the exposures $128^m$ and $80^m$, using the relation found for Pleiades,

$$m_{phot} = a - 10.6 \log D .$$

The scale was fixed by means of Krüger 60; its visual magnitude was assumed to be 9.2 (The value given in Adams and Joy's table), and its color-index to + 1.7. The visual magnitudes of the other stars were then determined by subtracting the color-index corresponding to the inferred spectral type.

Finally the parallaxes were computed by means of $M$ and $m$; we have $5 \log \pi = M - m - 5$. Our values of $M$ are of course very uncertain and only rough estimates, the parallaxes are therefore given with every reservation. The results are given in table 15. The positions of the stars are those of Bonner Durchm., for the unknown stars rough positions have been derived in order to help the identification of the stars.

Table 15.

| Star | | $m$ | R. A. 1855 | Decl. 1855 | Spectral type | $M$ | $\pi$ |
|---|---|---|---|---|---|---|---|
| | B. D. | | | | | | |
| 1 | $57^0 2514$ | 10.4 | $22^h 17.21^m$ | $57^0 13'.6$ | G 2 | + 8 | $0''.03$ |
| 2 | $56^0 2768$ | 10.2 | 18.28 | 56 16.2 | A | | |
| 3 | — | 11.1 | 18.4 | 57 9.5 | G – K | + 8 | 0.02 |
| 4 | $57^0 2519$ | 10.0 | 18.53 | 57 8.7 | F 3 | + 1.5 | 0.002 |
| 5 | $57^0 2521$ | 7.4 | 18.74 | 57 2.2 | K 2 | + 1 | 0.005 |
| 6 | — | 10.9 | 18.7 | 56 42 | F | | |
| 7 | — | 10.7 | 18.8 | 56 20 | F 7 | + 5 | 0.007 |
| 8 | $56^0 2772$ | 9.9 | 19.08 | 56 22.4 | K 2 | – 0.5 | 0.0008 |
| 9 | $56^0 2773$ | 10.0 | 19.46 | 56 45.7 | A 0 | | |
| 10 | — | 10.4 | 19.6 | 56 39 | F | | |
| 11 | — | 10.7 | 20.0 | 56 53 | F | | |
| 12 | — | 11.0 | 20.3 | 57 2 | A 0 | | |
| 13 | $57^0 2528$ | 10.3 | 20.46 | 57 5.9 | F 3 | + 1.5 | 0.002 |
| 14 | $57^0 2529$ | 10.8 | 20.47 | 57 3.6 | A 0 | | |
| 15 | — | 11.6 | 21.0 | 56 29 | B (?) | | |
| 16 | — | 11.3 | 21.3 | 56 18 | F 3 p | 0 | 0.0006 |
| 17 | — | 11.9 | 21·3 | 56 43 | B | | |
| 18 | — | 11.3 | 21.4 | 56 55.5 | F 1 | + 1.5 | 0.001 |
| 19 | $56^0 2777$ | 9.1 | 21.49 | 56 54.8 | K 2 | + 1 | 0.002 |

| | Star | $m$ | R. A. 1855 | Decl. 1855 | Spectral type | $M$ | $\pi$ |
|---|---|---|---|---|---|---|---|
| | B. D. | | | | | | |
| 20 | — | 10.2 | $22^h21.6^m$ | 56°23′ | F | | |
| 21 | 57°2534 | 10.5 | 21.80 | 57 4.9 | F | | |
| 22 | — | 10.5 | 21.8 | 56 40 | F 4 p | 0 | 0″.0008 |
| 23 | — | 11.2 | 21.8 | 56 14 | F 8 p | 0 | 0.0006 |
| 24 | — | 10.6 | 22.0 | 56 35 | F | | |
| 25 | — | 10.6 | 22.15 | 56 38 | F 6 | + 5 | 0.008 |
| 26 | 56°2780 | 8.7 | 22.19 | 56 33.8 | G 0 | + 3 | 0.007 |
| 27 | 56°2779 | 10.8 | 22.18 | 56 22.6 | K 8 | + 9 | 0.04 |
| 28 | — | 10.4 | 22.5 | 56 53 | F | | |
| 29 | 56°2782 | 9.8 | 22.62 | 56 20.9 | G 1 | 0 | 0.001 |
| 30 | — | 9.5 | 22.75 | 56 28 | G—K | + 6 | 0.02 |
| 31 | — | 9.6 | 22.8 | 56 46 | K 2 | + 1 | 0.002 |
| 32 | Krüger 60 | 9.2 | 22.83 | 56 58.3 | (M b) | (+11.2) | (0.″251) |
| 33 | — | 10.4 | 23.0 | 56 45 | K 2 | − 0.5 | 0.0007 |
| 34 | 56°2785 | 6.8 | 23.12 | 56 10.9 | K 4 | + 1 | 0.007 |
| 35 | 57°2544 | 10.4 | 23.25 | 57 8.2 | F 8 | + 4 | 0.005 |
| 36 | 56°2786 | 10.3 | 23.66 | 56 23.5 | K 3 | − 0.5 | 0.0007 |
| 37 | 56°2787 | 8.4 | 23.70 | 56 28.2 | K 5 | − 1 | 0.001 |
| 38 | 56°2788 | 8.6 | 23.90 | 56 58.6 | K 5 | + 1 | 0.003 |
| 39 | 56°2789 | 9.1 | 24.28 | 56 42.4 | F 7 | + 5 | 0.02 |

In addition to Krüger 60 there are two stars observed in B. D., 57°2514 and 56°2779, which are found here to be extreme dwarfs with considerable parallaxes. Mr K. Lundmark has kindly gone over a great number of star catalogues in order to find observations for these stars but without any result. From the positions given in B. D. the two stars seem to have a proper motion of about $\frac{1''}{2}$ per annum in the direction towards increasing R. A.; the present order between the neighbouring stars 57°2514 and 57°2516 and between 56°2779 and 56°2780 is opposite to that recorded in B. D. How much significance ought to be ascribed to this fact is perhaps doubtful, but it gives some support to the order of magnitude of the parallaxes. Between the stars 57°2511, 2512 and 57°2514 there are two neighbouring faint stars with large values of $\lambda_c$. We are probably concerned here with a system of two extreme dwarfs, connected with each others.

Let us finally consider the giants. There are three »yellow» giants here, for which I have given the spectral types F 3 p—F 8 p because of their abnormally high values of $\lambda_{\min}$. Possibly we are here concerned with stars of *c*-characteristics; I have estimated their absolute magnitudes to about 0 in accordance with the results of ADAMS and JOY for *c* and *ac* stars. In this way we get for these stars parallaxes of the same order of magnitude as for several »red» giants. Now the region examined here is projected upon a small star-cloud connected with the Milky Way in Cepheus. If we assume that the »yellow» and »red» giants mentioned are the most luminous objects of the star-cloud in question we may estimate the parallax for the Milky Way in Cepheus to 0''.0007. corresponding the the distance 1400 parsec or 4700 light-years. This must probably be considered as a minimum value of the distance.

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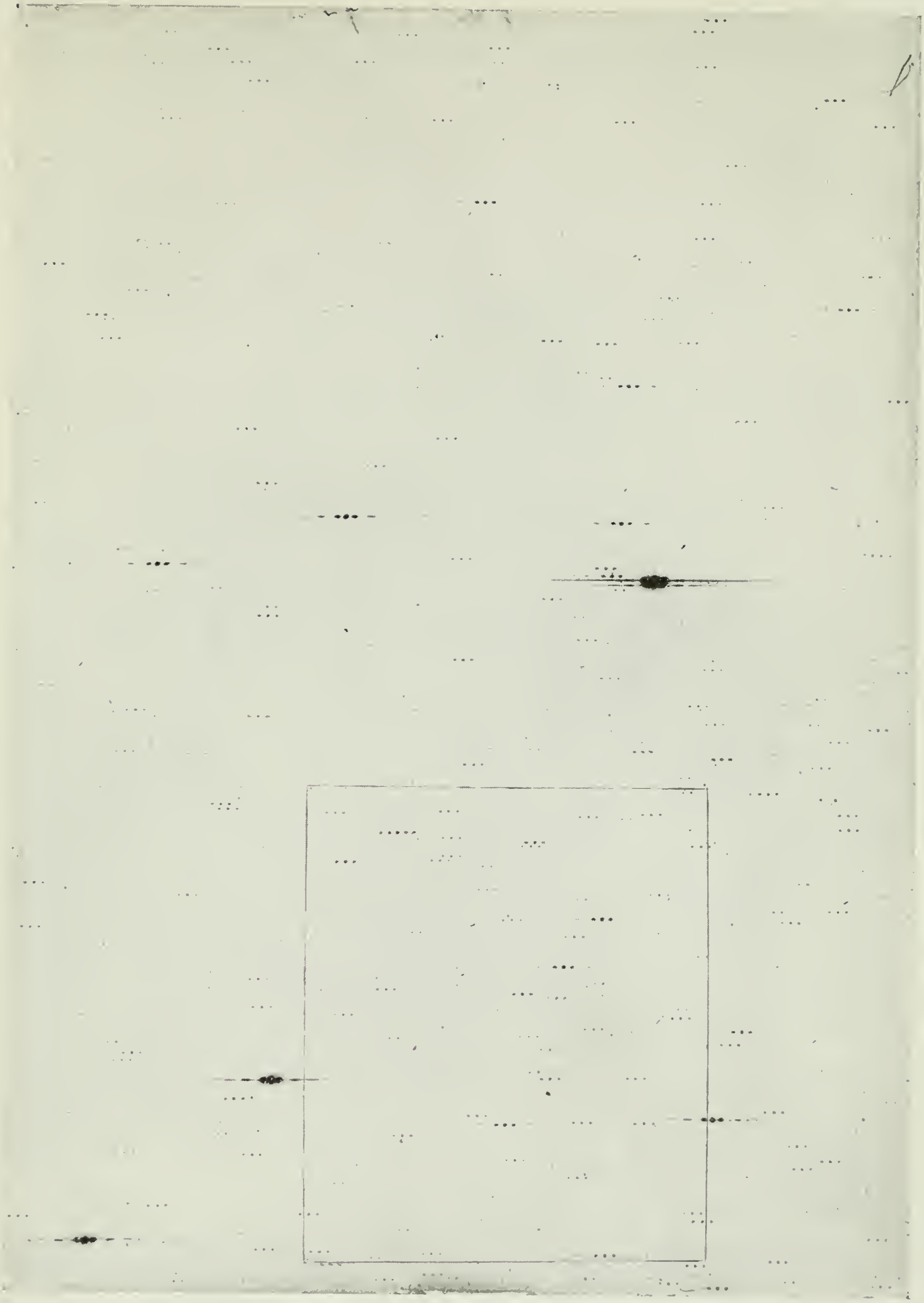

Grating spectra of stars in Cepheus, second plate, exposure $2^h\ 8^m$. Enlargement 2.2 : 1; 1 mm = 1′.1. The square marks the region examined here. The strongest star on the plate is δ Cephei.